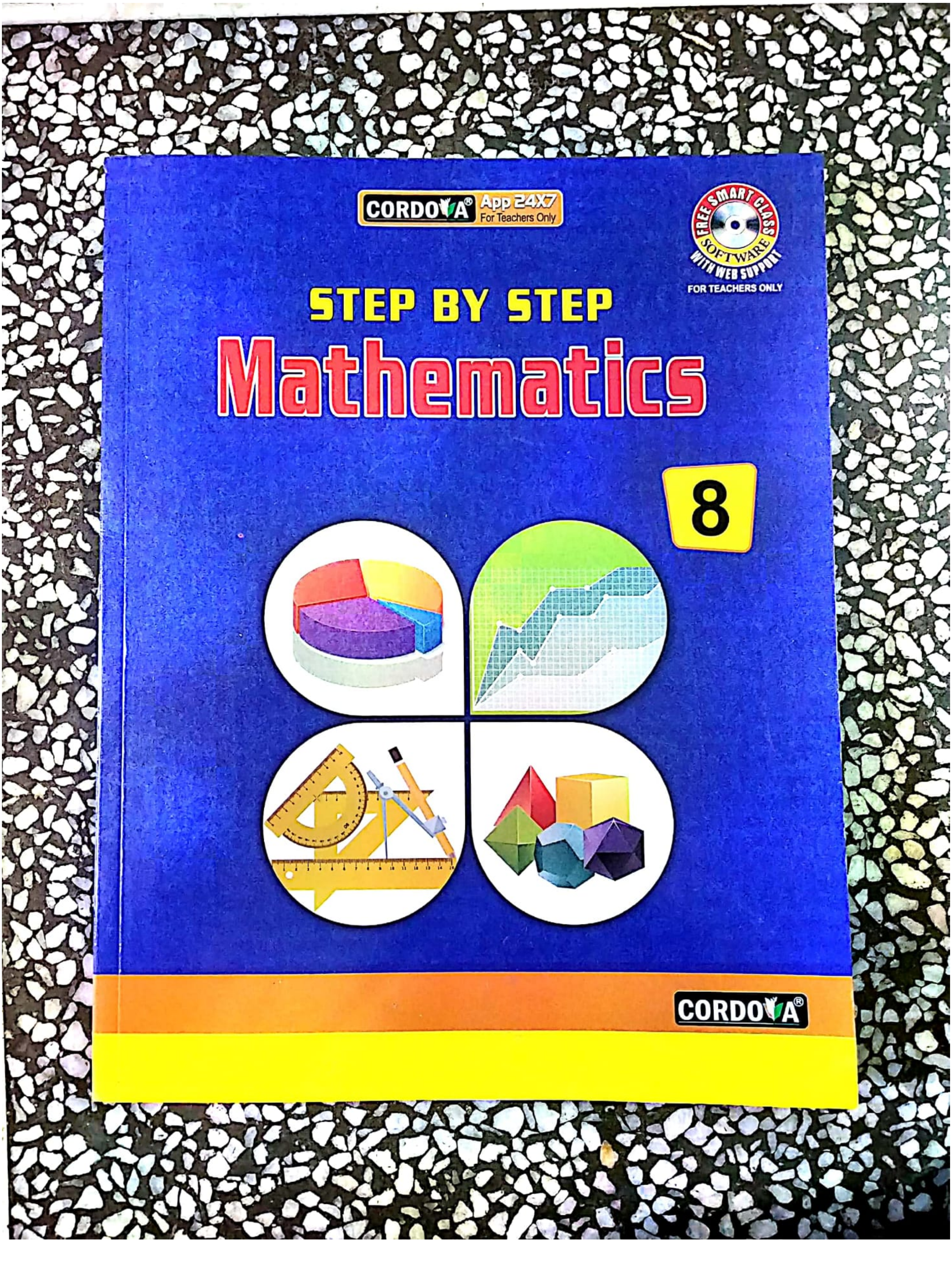
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STEP BY STEP
Mathematics
8
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CONTENTS

1 Rational Numbers

Use Cordova Smart Class Software on the smart board in class to learn about rational numbers.

RATIONAL NUMBERS

The numbers which can be written in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called **rational numbers**.

For example : $\frac{3}{5}, \frac{-2}{7}, \frac{4}{-11}$ and $\frac{-5}{-8}$ are rational numbers.

(*i*) **Positive rational numbers :** A rational number is said to be positive, if its numerator and denominator are either both positive or both negative.

For example : $\frac{7}{3}, \frac{-3}{-7}, \frac{2}{3}, \frac{-4}{-7}$ are positive rational numbers.

(*ii*) **Negative rational numbers :** A rational number is said to be negative, if either numerator or denominator is negative.

For example : $\frac{-5}{9}, \frac{3}{-7}, \frac{-4}{5}, \frac{2}{-3}$ are negative rational numbers.

0 can also be written as $\frac{0}{1} = \frac{0}{2} = \frac{0}{3}$.

Also, $1 = \frac{1}{1}, 2 = \frac{2}{1}, \ldots$ and $-1 = \frac{-1}{1}, -2 = \frac{-2}{1}, \ldots$.

Hence, all integers are rational numbers.

Equivalent Rational Numbers

(*i*) If $\frac{p}{q}$ is a rational number and m is a non-zero integer, then $\frac{p}{q} = \frac{p \times m}{q \times m}$.

For example :

$$\frac{-3}{5} = \frac{-3 \times 2}{5 \times 2} = \frac{-3 \times 3}{5 \times 3} = \frac{-3 \times 4}{5 \times 4}$$

$$\Rightarrow \frac{-3}{5} = \frac{-6}{10} = \frac{-9}{15} = \frac{-12}{20}$$

(*ii*) If $\frac{p}{q}$ is a rational number and m is a common divisor of p and q, then $\frac{p}{q} = \frac{p \div m}{q \div m}$.

For example : $\frac{36}{45} = \frac{36 \div 9}{45 \div 9} = \frac{4}{5}$

Similarly, $\frac{35}{49} = \frac{35 \div 7}{49 \div 7} = \frac{5}{7}$

The equivalent rational numbers are numbers that have same value but represented differently.

Standard Form of a Rational Number

A rational number $\frac{p}{q}$ is said to be in standard form, if denominator q is positive and p and q have no common divisor other than 1.

Example 1 : Express each of the following rational numbers in standard form :

(i) $\frac{30}{-72}$ (ii) $\frac{95}{105}$ (iii) $\frac{-65}{104}$ (iv) $\frac{-54}{-81}$

Solution : (i) $\frac{30}{-72} = \frac{-30}{72} = \frac{(-30) \div 6}{72 \div 6} = \frac{-5}{12}$ ($\because$ H.C.F. of 30 and 72 is 6)

(ii) $\frac{95}{105} = \frac{95 \div 5}{105 \div 5} = \frac{19}{21}$ ($\because$ H.C.F. of 95 and 105 is 5)

(iii) $\frac{-65}{104} = \frac{-65 \div 13}{104 \div 13} = \frac{-5}{8}$ ($\because$ H.C.F. of 65 and 104 is 13)

(iv) $\frac{-54}{-81} = \frac{54}{81} = \frac{54 \div 27}{81 \div 27} = \frac{2}{3}$ ($\because$ H.C.F. of 54 and 81 is 27)

Example 2 : Write down the rational number whose numerator is –3 and denominator is 4.

Solution :

Numerator $(p) = -3$

Denominator $(q) = 4$

Rational number $= \frac{p}{q} = \frac{-3}{4}$

Hence, the required rational number is $\frac{-3}{4}$.

Example 3 : Express $\frac{-2}{3}$ as a rational number with :

(i) numerator 4 (ii) denominator 9

Solution : (i) $\frac{-2}{3} = \frac{-2 \times (-2)}{3 \times (-2)} = \frac{4}{-6}$ (ii) $\frac{-2}{3} = \frac{-2 \times 3}{3 \times 3} = \frac{-6}{9}$

Example 4 : Write three rational numbers equivalent to the following rational numbers :

(i) $\frac{4}{3}$ (ii) $\frac{-6}{7}$

Solution : (i) $\frac{4}{3} = \frac{4 \times 2}{3 \times 2} = \frac{8}{6}$; $\frac{4}{3} = \frac{4 \times 3}{3 \times 3} = \frac{12}{9}$; $\frac{4}{3} = \frac{4 \times 4}{3 \times 4} = \frac{16}{12}$

Hence, three rational numbers equivalent to $\frac{4}{3}$ are $\frac{8}{6}, \frac{12}{9}$ and $\frac{16}{12}$.

(ii) $\frac{-6}{7} = \frac{-6 \times 2}{7 \times 2} = \frac{-12}{14}$; $\frac{-6}{7} = \frac{-6 \times 3}{7 \times 3} = \frac{-18}{21}$; $\frac{-6}{7} = \frac{-6 \times 4}{7 \times 4} = \frac{-24}{28}$

Hence, three rational numbers equivalent to $\frac{-6}{7}$ are $\frac{-12}{14}, \frac{-18}{21}$ and $\frac{-24}{28}$.

Absolute Value of a Rational Number

It is the numerical value of a rational number.

To find the absolute value of a rational number, we take the absolute value of numerator and absolute value of denominator.

If $\frac{p}{q}$ is a rational number, then absolute value of $\frac{p}{q}$ is written as $\left|\frac{p}{q}\right| = \frac{|p|}{|q|}$.

It is either zero or positive.

Example 5 : Find the absolute value of the following rational numbers :

(i) $\frac{5}{12}$ (ii) $\frac{-8}{7}$ (iii) $\frac{3}{-5}$ (iv) $\frac{-4}{-3}$

Solution : (i) $\left|\frac{5}{12}\right| = \frac{|5|}{|12|} = \frac{5}{12}$ (ii) $\left|\frac{-8}{7}\right| = \frac{|-8|}{|7|} = \frac{8}{7}$

(iii) $\left|\frac{3}{-5}\right| = \frac{|3|}{|-5|} = \frac{3}{5}$ (iv) $\left|\frac{-4}{-3}\right| = \frac{|-4|}{|-3|} = \frac{4}{3}$

EXERCISE 1.1

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Which of the following are rational numbers?

   $\frac{0}{5}, 2, \frac{3}{4}, \frac{-1}{\sqrt{2}}, \frac{5}{0}$

2. Identify the numerator and denominator in the following rational numbers :

   $\frac{-2}{3}, \frac{4}{1}, \frac{0}{3}, 5, \frac{3}{-1}$

3. Express $\frac{-5}{3}$ as a rational number whose numerator is :

   (i) 5 (ii) 35 (iii) –20 (iv) 15

4. Express $\frac{3}{4}$ as a rational number whose denominator is :

   (i) 12 (ii) 20 (iii) –16 (iv) 28

5. Express in standard form :

   (i) $\frac{15}{65}$ (ii) $\frac{33}{-77}$ (iii) $\frac{-13}{-78}$ (iv) $\frac{-21}{15}$

6. Write down the absolute value of the following rational numbers :

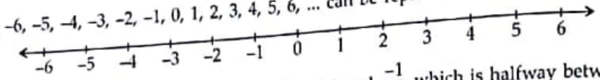
   (i) $\frac{3}{-5}$ (ii) $\frac{-4}{7}$ (iii) $\frac{8}{9}$ (iv) $\frac{-6}{-11}$

7. Write three rational numbers equivalent to the following rational numbers :

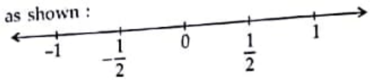
   (i) $\frac{-2}{3}$ (ii) $\frac{3}{5}$ (iii) $\frac{7}{-6}$

REPRESENTATION OF RATIONAL NUMBERS ON THE NUMBER LINE

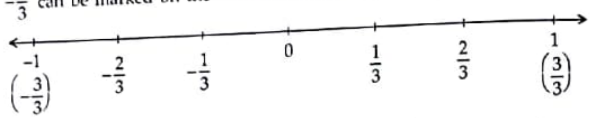
The integers ..., –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, ... can be represented on the number line as shown :



The rational number $\frac{1}{2}$ which is halfway between 0 and 1 and $\frac{-1}{2}$ which is halfway between 0 and –1 can be shown on the number line as shown :



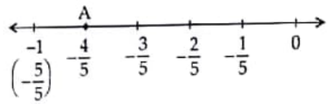
For representation of rational number $\frac{1}{3}$, we divide the distance between 0 and 1 into three equal parts and for $-\frac{1}{3}$, we divide the distance between 0 and –1 (towards left of zero) into three equal parts. The rational numbers $\frac{1}{3}$ and $-\frac{1}{3}$ can be marked on the number line as shown below :



The rational number $\frac{2}{3}$ lies between $\frac{1}{3}$ and 1, whereas $-\frac{2}{3}$ lies between –1 and $-\frac{1}{3}$.

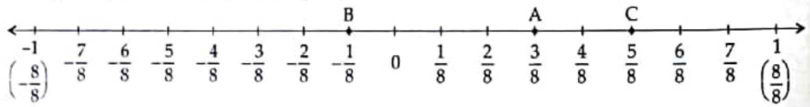
Example 6 : Represent $-\frac{4}{5}$ on the number line.

Solution : To represent rational number $-\frac{4}{5}$, we divide the distance between 0 and –1 into five equal parts. Starting from zero, move towards –1 (left) and the 4th mark will represent $-\frac{4}{5}$ as shown below :



Example 7 : Represent $\frac{3}{8}$, $-\frac{1}{8}$ and $\frac{5}{8}$ on the number line.

Solution : We divide the distance between 0 and 1 into 8 equal parts and the distance between 0 and –1 (towards left of zero) into 8 equal parts.



Moving towards right of zero, first mark will be $\frac{1}{8}$, second $\frac{2}{8}$ and so on.

Similarly, moving towards left of zero, first mark will be $-\frac{1}{8}$, second $-\frac{2}{8}$ and so on.

Thus, the points A, B and C represent $\frac{3}{8}$, $-\frac{1}{8}$ and $\frac{5}{8}$ on the number line respectively.

COMPARISON OF RATIONAL NUMBERS

We know that,

(*i*) Every positive rational number is greater than zero, e.g. $\frac{3}{5} > 0$.

(*ii*) Every negative rational number is less than zero, e.g. $-\frac{5}{9} < 0$.

Method of Comparing Rational Numbers

Step 1 : Express the rational numbers with positive denominators.

Step 2 : Take L.C.M. of the positive denominators.

Step 3 : Express each rational number with L.C.M. as common denominator.

Step 4 : Compare the numerators. The rational number having the greater numerator is greater.

Example 8 : Compare the following rational numbers :

(*i*) $\frac{3}{5}$ and $\frac{2}{3}$ (*ii*) $\frac{-3}{7}$ and $\frac{5}{-4}$

Solution : (*i*) The two rational numbers are $\frac{3}{5}$ and $\frac{2}{3}$.

The L.C.M. of denominators 5 and 3 is 15.

We make denominator of each rational number equal to 15.

$\frac{3}{5} = \frac{3\times3}{5\times3} = \frac{9}{15}$; $\frac{2}{3} = \frac{2\times5}{3\times5} = \frac{10}{15}$

On comparing the numerators of $\frac{9}{15}$ and $\frac{10}{15}$, we find that $9 < 10$.

Therefore, $\frac{9}{15} < \frac{10}{15}$

$\Rightarrow$ $\frac{3}{5} < \frac{2}{3}$

(*ii*) The two rational numbers are $\frac{-3}{7}$ and $\frac{5}{-4}$.

Making the denominators positive, we get

$\frac{-3}{7}$ and $\frac{-5}{4}$ (Standard form)

Now, L.C.M. of 7 and 4 is 28, so, we make the denominator of each rational number equal to 28.

$\therefore$ $\frac{-3}{7} = \frac{-3\times4}{7\times4} = \frac{-12}{28}$; $\frac{-5}{4} = \frac{-5\times7}{4\times7} = \frac{-35}{28}$

Now, compare the numerators.

Since, $-12 > -35$

therefore $\frac{-12}{28} > \frac{-35}{28}$ $\Rightarrow$ $\frac{-3}{7} > \frac{-5}{4}$

$\Rightarrow$ $\frac{-3}{7} > \frac{5}{-4}$

We can also compare the rational numbers by using **Cross Multiplication Method.**

Example 9 : Compare the following :

(*i*) $\frac{-5}{8}, \frac{-3}{7}$ (*ii*) $\frac{-7}{5}, \frac{-11}{8}$

Solution : (i) The two rational numbers are $\frac{-5}{8}$ and $\frac{-3}{7}$.

Using cross multiplication, $\frac{-5}{8} \times \frac{-3}{7}$

$-5 \times 7 \quad -3 \times 8$

$-35 \quad -24$

Since, $-35 < -24$

$\therefore \quad \frac{-5}{8} < \frac{-3}{7}$

(ii) The given rational numbers are $\frac{-7}{5}$ and $\frac{-11}{8}$.

Using cross multiplication, $\frac{-7}{5} \times \frac{-11}{8}$

$-7 \times 8 \quad -11 \times 5$

$-56 \quad -55$

Since, $-56 < -55$

$\therefore \quad \frac{-7}{5} < \frac{-11}{8}$

Example 10 : Arrange the given rational numbers in ascending order : $\frac{-7}{4}, \frac{5}{6}, \frac{7}{-12}$

Solution : The given rational numbers are $\frac{-7}{4}, \frac{5}{6}$ and $\frac{7}{-12}$.

Writing the given rational numbers in standard form, we get $\frac{-7}{4}, \frac{5}{6}, \frac{-7}{12}$.

L.C.M. of 4, 6 and 12 is 12.

$\therefore \quad \frac{-7}{4} = \frac{-7 \times 3}{4 \times 3} = \frac{-21}{12}\ ;\ \frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}\ ;\ \frac{-7}{12}$

Now, comparing the numerators, we get, $-21 < -7 < 10$

$\therefore \quad \frac{-21}{12} < \frac{-7}{12} < \frac{10}{12}$ or $\frac{-7}{4} < \frac{7}{-12} < \frac{5}{6}$

EXERCISE 1.2

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Which of the rational numbers is greater in each of the following :

   (i) $\frac{3}{4}$ and 0 (ii) $\frac{-1}{2}$ and $\frac{4}{-7}$ (iii) $\frac{8}{15}$ and $\frac{3}{10}$ (iv) $\frac{-1}{2}$ and $\frac{8}{-5}$

2. Represent the following rational numbers on the number line :

   (i) $\frac{-5}{6}$ (ii) $\frac{3}{7}$ (iii) $\frac{-2}{7}$ (iv) $\frac{-8}{11}$

3. Arrange the following rational numbers in ascending order :

   (i) $\frac{8}{-15}, \frac{-3}{10}, \frac{-13}{20}, \frac{17}{-30}$ (ii) $\frac{-13}{5}, -2, \frac{7}{-3}, \frac{2}{3}$

4. Arrange the following rational numbers in descending order :

   (i) $\frac{-5}{12}, \frac{-7}{6}, \frac{3}{-8}, \frac{-11}{7}$ (ii) $\frac{-17}{11}, \frac{7}{-5}, \frac{-11}{9}, \frac{13}{-8}$

5. Use correct sign <, = or > :

   (i) $\frac{-2}{13}$ ☐ $\frac{3}{-7}$ (ii) $\frac{-13}{6}$ ☐ -2 (iii) $\frac{-3}{2}$ ☐ $\frac{6}{-5}$

   (iv) $\frac{-3}{10}$ ☐ $\frac{6}{-20}$ (v) 0 ☐ $\frac{-2}{-3}$ (vi) $\frac{-7}{12}$ ☐ $\frac{-13}{9}$

6. Write any five rational numbers which are smaller than –1.

7. Write any five rational numbers which are greater than $\frac{-3}{2}$.

ADDITION OF RATIONAL NUMBERS

The addition of rational numbers is carried out in the same way as that of the addition of fractions. Before adding rational numbers, we make their denominators positive.

Adding rational numbers with same denominator

Let the two rational numbers be $\frac{a}{c}$ and $\frac{b}{c}$ having common denominator c.

On adding both the rational numbers,

we get $\quad \frac{a}{c} + \frac{b}{c} = \frac{(a+b)}{c}$

To add the rational numbers having same denominator, we follow the following steps :

Step 1 : Add the numerators.

Step 2 : Write a rational number whose numerator is the sum obtained in Step 1 and whose denominator is the common denominator of the given rational numbers.

Example 11 : Find the sum :

(i) $\frac{6}{11} + \frac{(-15)}{11}$ (ii) $\frac{6}{-7} + \frac{8}{7}$ (iii) $\frac{-4}{9} + \frac{(-6)}{9}$

Solution : (i) $\frac{6}{11} + \frac{(-15)}{11} = \frac{6+(-15)}{11} = \frac{-9}{11}$ (ii) $\frac{6}{-7} + \frac{8}{7} = \frac{-6}{7} + \frac{8}{7} = \frac{(-6)+8}{7} = \frac{2}{7}$

(iii) $\frac{-4}{9} + \frac{(-6)}{9} = \frac{-4+(-6)}{9} = \frac{-10}{9}$

Adding rational numbers with different denominators

We follow the following steps :

Step 1 : If the denominators of the given rational numbers are negative, make them positive.

Step 2 : Find the L.C.M. of denominators.

Step 3 : Express each number with the L.C.M. as common denominator.

Step 4 : Write a rational number whose numerator is the sum of the numerators and whose denominator is the L.C.M. obtained in Step 3.

Example 12 : Find the sum :

(i) $\frac{6}{13}$ and $\frac{-4}{39}$ (ii) $\frac{-16}{9}$ and $\frac{-7}{12}$ (iii) $\frac{4}{3}$ and $\frac{5}{-7}$

Solution : (i) The L.C.M. of denominators 13 and 39 is 39.

$$\frac{6}{13} = \frac{6\times3}{13\times3} = \frac{18}{39};\ \frac{-4}{39}$$

$$\therefore\ \frac{6}{13} + \left(\frac{-4}{39}\right) = \frac{18}{39}+\left(\frac{-4}{39}\right) = \frac{18-4}{39} = \frac{14}{39}$$

(ii) The L.C.M. of denominators 9 and 12 is 36.

$$\frac{-16}{9} = \frac{-16\times4}{9\times4} = \frac{-64}{36};\ \frac{-7}{12} = \frac{-7\times3}{12\times3} = \frac{-21}{36}$$

$$\therefore\ \left(\frac{-16}{9}\right) + \left(\frac{-7}{12}\right) = \left(\frac{-64}{36}\right) + \left(\frac{-21}{36}\right) = \frac{-64-21}{36} = \frac{-85}{36}$$

(iii) The L.C.M. of denominators 3 and 7 is 21.

$$\therefore\ \frac{4}{3} = \frac{4\times7}{3\times7} = \frac{28}{21};\ \frac{5}{-7} = \frac{-5}{7} = \frac{-5\times3}{7\times3} = \frac{-15}{21}$$

$$\therefore\ \frac{4}{3}+\left(\frac{5}{-7}\right) = \frac{28}{21}+\left(\frac{-15}{21}\right) = \frac{28-15}{21} = \frac{13}{21}$$

Properties of Addition of Rational Numbers

1. Closure Property

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\left(\frac{a}{b}+\frac{c}{d}\right)$ is also a rational number.

Examples : (i) Consider the rational numbers $\frac{1}{3}$ and $\frac{4}{9}$, then

$$\frac{1}{3}+\frac{4}{9} = \left(\frac{1\times3}{3\times3}\right)+\frac{4}{9} = \frac{3}{9}+\frac{4}{9} = \frac{3+4}{9} = \frac{7}{9}$$

which is a rational number.

(ii) Consider the rational numbers $\frac{3}{4}$ and $\frac{-5}{8}$, then

$$\frac{3}{4}+\left(\frac{-5}{8}\right) = \frac{3\times2}{4\times2}+\left(\frac{-5}{8}\right) = \frac{6}{8}+\left(\frac{-5}{8}\right) = \frac{6-5}{8} = \frac{1}{8}$$

which is a rational number.

(iii) Consider the rational numbers $\frac{-5}{12}$ and $\frac{-3}{4}$, then

$$\left(\frac{-5}{12}\right)+\left(\frac{-3}{4}\right) = \left(\frac{-5}{12}\right)+\left(\frac{-3\times3}{4\times3}\right) = \frac{-5}{12}+\left(\frac{-9}{12}\right) = \frac{-5+(-9)}{12} = \frac{-14}{12} = \frac{-7}{6}$$

which is a rational number.

2. Commutative Property

Two rational numbers can be added in any order. If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then

$$\left(\frac{a}{b}+\frac{c}{d}\right) = \left(\frac{c}{d}+\frac{a}{b}\right)$$

Examples : (i) Consider the rational numbers $\frac{3}{4}$ and $\frac{5}{6}$,

then, $\frac{3}{4}+\frac{5}{6} = \frac{3\times3}{4\times3}+\frac{5\times2}{6\times2} = \frac{9}{12}+\frac{10}{12} = \frac{9+10}{12} = \frac{19}{12}$

Also, $\frac{5}{6}+\frac{3}{4} = \frac{5\times2}{6\times2}+\frac{3\times3}{4\times3} = \frac{10}{12}+\frac{9}{12} = \frac{19}{12}$

Hence, $\frac{3}{4}+\frac{5}{6} = \frac{5}{6}+\frac{3}{4}$

(ii) Consider the rational numbers $\frac{5}{6}$ and $\frac{-4}{9}$,

then, $\frac{5}{6}+\left(\frac{-4}{9}\right) = \frac{5\times3}{6\times3}+\frac{(-4\times2)}{9\times2} = \frac{15}{18}+\left(\frac{-8}{18}\right) = \frac{15-8}{18} = \frac{7}{18}$

Also, $\left(\frac{-4}{9}\right)+\frac{5}{6} = \frac{(-4\times2)}{9\times2}+\frac{5\times3}{6\times3} = \frac{-8}{18}+\frac{15}{18} = \frac{-8+15}{18} = \frac{7}{18}$

Hence, $\frac{5}{6}+\left(\frac{-4}{9}\right) = \frac{-4}{9}+\frac{5}{6}$

(iii) Consider the rational numbers $\frac{-4}{3}$ and $\frac{-3}{5}$,

then, $\left(\frac{-4}{3}\right)+\left(\frac{-3}{5}\right) = \frac{(-4\times5)}{3\times5}+\frac{(-3\times3)}{5\times3} = \frac{-20}{15}+\left(\frac{-9}{15}\right) = \frac{-20-9}{15} = \frac{-29}{15}$

Also, $\left(\frac{-3}{5}\right)+\left(\frac{-4}{3}\right) = \frac{(-3\times3)}{(5\times3)}+\frac{(-4\times5)}{3\times5} = \frac{-9}{15}+\left(\frac{-20}{15}\right) = \frac{-9-20}{15} = \frac{-29}{15}$

Hence, $\left(\frac{-4}{3}\right)+\left(\frac{-3}{5}\right) = \left(\frac{-3}{5}\right)+\left(\frac{-4}{3}\right)$.

3. Associative Property

While adding three rational numbers, they can be grouped in any order.

If $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$ are three rational numbers, then $\left(\frac{a}{b}+\frac{c}{d}\right)+\frac{e}{f} = \frac{a}{b}+\left(\frac{c}{d}+\frac{e}{f}\right)$

Example : Consider the rational numbers $\frac{-3}{4}, \frac{5}{6}$ and $\frac{2}{9}$.

Then, $\left(\frac{-3}{4}+\frac{5}{6}\right)+\frac{2}{9} = \left(\frac{-9}{12}+\frac{10}{12}\right)+\frac{2}{9} = \frac{1}{12}+\frac{2}{9} = \frac{3}{36}+\frac{8}{36} = \frac{11}{36}$

Also, $\frac{-3}{4}+\left(\frac{5}{6}+\frac{2}{9}\right) = \frac{-3}{4}+\left(\frac{15}{18}+\frac{4}{18}\right) = \frac{-3}{4}+\frac{19}{18} = \frac{-27}{36}+\frac{38}{36} = \frac{11}{36}$

Hence, $\left(\frac{-3}{4}+\frac{5}{6}\right)+\frac{2}{9} = \frac{-3}{4}+\left(\frac{5}{6}+\frac{2}{9}\right)$

4. Existence of Additive Identity

If $\frac{a}{b}$ is a rational number, then $\frac{a}{b}+0=0+\frac{a}{b}=\frac{a}{b}$.

0 is called the **additive identity** for rational numbers.

Examples : (*i*) $\frac{3}{7}+0=0+\frac{3}{7}=\frac{3}{7}$ (*ii*) $\frac{-4}{5}+0=0+\left(\frac{-4}{5}\right)=\frac{-4}{5}$

5. Existence of Additive Inverse

If $\frac{a}{b}$ is a rational number, then there exists a rational number $\frac{-a}{b}$ such that $\frac{a}{b}+\left(\frac{-a}{b}\right)=0=\frac{-a}{b}+\frac{a}{b}$.

$\frac{a}{b}$ and $\frac{-a}{b}$ are called **additive inverse** of each other.

Example : Consider the rational numbers $\frac{3}{7}$ and $\frac{-3}{7}$.

Then, $\frac{3}{7}+\left(\frac{-3}{7}\right)=\left(\frac{-3}{7}\right)+\frac{3}{7}=0$

Thus, $\frac{3}{7}$ and $\frac{-3}{7}$ are additive inverse of each other.

SUBTRACTION OF RATIONAL NUMBERS

The difference of two rational numbers $\frac{a}{b}$ and $\frac{c}{d}$ is defined as $\frac{a}{b}-\frac{c}{d}=\frac{ad-bc}{bd}$.

Example 13 : Subtract :

(*i*) $\frac{6}{7}$ from $\frac{4}{5}$ (*ii*) $\frac{-8}{9}$ from $\frac{-3}{5}$ (*iii*) $\frac{5}{6}$ from $\frac{-2}{3}$

Solution : (*i*) $\frac{4}{5}-\frac{6}{7}=\frac{28-30}{35}=\frac{-2}{35}$

(*ii*) $\frac{-3}{5}-\left(\frac{-8}{9}\right)=\frac{-3}{5}+\frac{8}{9}=\frac{-27+40}{45}=\frac{13}{45}$

(*iii*) $\frac{-2}{3}-\frac{5}{6}=\frac{-12-15}{18}=\frac{-27}{18}=\frac{-3}{2}$

Properties of Subtraction of Rational Numbers

1. Closure Property

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b}-\frac{c}{d}$ is a rational number *i.e.,* difference of two rational numbers is a rational number.

Examples : (*i*) Consider the two rational numbers $\frac{7}{3}$ and $\frac{3}{4}$.

Then, $\frac{7}{3}-\frac{3}{4}=\frac{28-9}{12}=\frac{19}{12}$, which is a rational number.

(*ii*) Consider the two rational numbers $\frac{-3}{5}$ and $\frac{2}{-3}$.

Then, $\frac{-3}{5}-\left(\frac{2}{-3}\right)=\frac{-3}{5}-\left(\frac{-2}{3}\right)=\frac{-3}{5}+\frac{2}{3}=\frac{-9+10}{15}=\frac{1}{15}$

which is a rational number.

2. Commutative Property

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\frac{a}{b}-\frac{c}{d}\neq\frac{c}{d}-\frac{a}{b}$ *i.e.,* subtraction of rational numbers is not commutative.

Example : If $\frac{3}{4}$ and $\frac{7}{8}$ are two rational numbers, then

$\frac{3}{4}-\frac{7}{8}=\left(\frac{3\times2}{4\times2}\right)-\frac{7}{8}=\frac{6}{8}-\frac{7}{8}=\frac{6-7}{8}=\frac{-1}{8}$

also, $\frac{7}{8}-\frac{3}{4}=\frac{7}{8}-\left(\frac{3\times2}{4\times2}\right)=\frac{7}{8}-\frac{6}{8}=\frac{7-6}{8}=\frac{1}{8}$

Hence, $\frac{3}{4}-\frac{7}{8}\neq\frac{7}{8}-\frac{3}{4}$.

Associative property, identity property and inverse property do not hold good under subtraction.

Let us take some more examples of addition and subtraction of rational numbers.

Example 14 : What should be added to $\frac{-7}{8}$ to get $\frac{-3}{2}$?

Solution : Let the number to be added be x.

Then, $\frac{-7}{8}+x=\frac{-3}{2}$

$\Rightarrow$ $x=\frac{7}{8}-\frac{3}{2}=\frac{7-12}{8}=\frac{-5}{8}$

Hence, $\frac{-5}{8}$ should be added.

Example 15 : What should be subtracted from $\frac{5}{7}$ to get $\frac{1}{21}$?

Solution : Let the number to be subtracted be x.

Then, $\frac{5}{7}-x=\frac{1}{21}$

$\Rightarrow$ $x=\frac{5}{7}-\frac{1}{21}=\left(\frac{5\times3}{7\times3}\right)-\frac{1}{21}=\frac{15}{21}-\frac{1}{21}$

$=\frac{15-1}{21}=\frac{14}{21}=\frac{2}{3}$

Hence, $\frac{2}{3}$ should be subtracted from $\frac{5}{7}$ to get $\frac{1}{21}$.

Example 16 : The sum of two rational numbers is $\frac{-6}{11}$. If one of the numbers is $\frac{-64}{77}$, find the other.

Solution : Let the other number be x.

Then $\frac{-64}{77}+x=\frac{-6}{11}$

$\Rightarrow$ $x=\frac{64}{77}-\frac{6}{11}=\frac{64}{77}-\left(\frac{6\times7}{11\times7}\right)=\frac{64}{77}-\frac{42}{77}=\frac{64-42}{77}=\frac{22}{77}=\frac{2}{7}$

Hence, the other rational number is $\frac{2}{7}$.

Example 17 : Find the sum by suitable arrangement : $\frac{3}{5}+\left(\frac{-2}{3}\right)+\left(\frac{-11}{5}\right)+\frac{4}{3}$

Solution : $\frac{3}{5}+\left(\frac{-2}{3}\right)+\left(\frac{-11}{5}\right)+\frac{4}{3} = \left[\frac{3}{5}+\left(\frac{-11}{5}\right)\right]+\left[\frac{4}{3}+\left(\frac{-2}{3}\right)\right] = \left(\frac{3-11}{5}\right)+\left(\frac{4-2}{3}\right)$

$= \frac{-8}{5}+\frac{2}{3} = \left(\frac{-8\times3}{5\times3}\right)+\left(\frac{2\times5}{3\times5}\right) = \frac{-24}{15}+\frac{10}{15} = \frac{-24+10}{15} = \frac{-14}{15}$

EXERCISE 1.3

Use Cordova Smart Class Software on the smart board in class to do Exercise.

1. Add :
   (i) $\frac{5}{8}$ and $\frac{3}{-10}$ (ii) $\frac{-3}{10}$ and $\frac{7}{-15}$ (iii) 4 and $\frac{5}{6}$ (iv) $\frac{15}{-7}$ and $\frac{8}{3}$
2. Subtract :
   (i) $\frac{8}{3}$ from $\frac{13}{7}$ (ii) $\frac{-4}{13}$ from $\frac{6}{-7}$ (iii) $\frac{11}{6}$ from $\frac{-2}{9}$ (iv) $\frac{-7}{10}$ from $\frac{2}{5}$
3. Simplify :
   (i) $\frac{5}{6}-\frac{3}{8}+\frac{7}{12}$ (ii) $\frac{11}{-18}-\frac{5}{16}+\frac{4}{9}$ (iii) $2+\left(\frac{-2}{3}\right)+\left(\frac{-4}{5}\right)$ (iv) $\frac{-9}{2}+\left(\frac{-8}{3}\right)+\frac{11}{6}$
4. If $x=\frac{3}{7}$ and $y=\frac{5}{3}$, verify that $x+y=y+x$. Name the property used.
5. If $x=\frac{4}{7}$, $y=\frac{-5}{21}$ and $z=\frac{1}{3}$, verify that $(x+y)+z=x+(y+z)$. Name the property used.
6. Subtract :
   (i) $\frac{3}{5}$ from $\frac{5}{6}$ (ii) $\frac{-5}{8}$ from $\frac{-4}{3}$
7. Express each of the following as a rational number :
   (i) $\frac{3}{7}+\left(\frac{-2}{9}\right)+\frac{7}{9}$ (ii) $\frac{7}{12}-\frac{5}{6}+\frac{1}{8}-\frac{5}{12}$ (iii) $\frac{-4}{3}-2+\frac{2}{5}+1$
8. Write the additive inverse of :
   (i) $-\frac{3}{7}$ (ii) $\frac{16}{-3}$ (iii) $\frac{7}{9}$ (iv) $-\left(\frac{11}{-5}\right)$
9. The sum of two numbers is $\frac{-5}{3}$. If one of the numbers is $\frac{-12}{3}$, what is the other number?
10. The sum of two numbers is $\frac{-23}{9}$. If one of the numbers is $\frac{5}{9}$, what is the other number?
11. What should be added to $\left(\frac{1}{3}+\frac{1}{4}+\frac{1}{6}\right)$ to get 1?
12. What should be subtracted from $\frac{3}{5}$ to get $\frac{5}{3}$?

13. What should be subtracted from $\left(\frac{3}{4}-\frac{1}{3}\right)$ to get $-\frac{1}{4}$?
14. By suitable arrangement find the sum :
    (i) $\frac{7}{3}+\left(\frac{-8}{5}\right)+\frac{3}{5}+\left(\frac{2}{-3}\right)$ (ii) $\frac{-9}{5}+\left(\frac{2}{-3}\right)+\frac{1}{5}+\frac{3}{5}$
15. Subtract the sum of $-\frac{5}{6}$ and $\frac{4}{5}$ from the sum of $\frac{-3}{5}$ and $\frac{7}{15}$.

MULTIPLICATION OF RATIONAL NUMBERS

If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}$.

i.e., The product of two rational numbers = $\frac{\text{Product of their numerators}}{\text{Product of their denominators}}$

Example 18 : Multiply :

(i) $\frac{3}{4}$ by $\frac{3}{7}$ (ii) $\frac{-4}{7}$ by $\frac{14}{3}$ (iii) $\frac{13}{5}$ by $\frac{-25}{91}$ (iv) $\frac{-8}{25}$ by $\frac{15}{16}$

Solution : (i) $\frac{3}{4}\times\frac{3}{7} = \frac{3\times3}{4\times7}=\frac{9}{28}$ (ii) $\frac{-4}{7}\times\frac{14}{3} = \frac{(-4)\times14}{7\times3}=\frac{-56}{21}=\frac{-8}{3}$

(iii) $\frac{13}{5}\times\left(\frac{-25}{91}\right) = \frac{13\times(-25)}{5\times91}=\frac{-5}{7}$ (iv) $\left(\frac{-8}{25}\right)\times\frac{15}{16} = \frac{(-8)\times15}{25\times16}=\frac{-3}{10}$

Example 19 : Simplify :

(i) $\left(\frac{1}{3}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{4}\right)$ (ii) $\left[\frac{-14}{5}\times\frac{20}{21}\right]-\left[\frac{15}{3}\times\left(\frac{-9}{2}\right)\right]$ (iii) $\left(-\frac{7}{9}\times\frac{15}{3}\right)-\left(2\times\frac{3}{4}\right)+\left(\frac{1}{3}\times\frac{7}{2}\right)$

Solution : (i) $\left(\frac{1}{3}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{4}\right) = \frac{1}{6}+\frac{1}{8} = \left(\frac{1\times4}{6\times4}\right)+\left(\frac{1\times3}{8\times3}\right) = \frac{4}{24}+\frac{3}{24} = \frac{4+3}{24}=\frac{7}{24}$

(ii) $\left[\frac{-14}{5}\times\frac{20}{21}\right]-\left[\frac{15}{3}\times\left(\frac{-9}{2}\right)\right] = \left(\frac{-14\times20}{5\times21}\right) - \left(-\frac{15\times9}{3\times2}\right)$

$= -\frac{8}{3}-\left(-\frac{45}{2}\right) = \frac{-8}{3}+\frac{45}{2}$

$= \left(\frac{-8\times2}{3\times2}\right)+\left(\frac{45\times3}{2\times3}\right) = \frac{-16}{6}+\frac{135}{6} = \frac{-16+135}{6}=\frac{119}{6}$

(iii) $\left(-\frac{7}{9}\times\frac{15}{3}\right)-\left(2\times\frac{3}{4}\right)+\left(\frac{1}{3}\times\frac{7}{2}\right) = -\left(\frac{7\times15}{9\times3}\right)-\left(\frac{2\times3}{4}\right)+\left(\frac{1\times7}{3\times2}\right)$

$= -\frac{35}{9}-\frac{3}{2}+\frac{7}{6} = \left(\frac{-35\times2}{9\times2}\right)-\left(\frac{3\times9}{2\times9}\right)+\left(\frac{7\times3}{6\times3}\right)$

$= \frac{-70}{18}-\frac{27}{18}+\frac{21}{18} = \frac{-70-27+21}{18}=\frac{-76}{18}=\frac{-38}{9}$

Properties of Multiplication of Rational Numbers

1. Closure Property

The product of two rational numbers is always a rational number.

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then $\left(\frac{a}{b}\times\frac{c}{d}\right)$ is also a rational number.

Examples : (*i*) Consider two rational numbers $\frac{2}{3}$ and $\frac{5}{7}$.

Then their product $\frac{2}{3}\times\frac{5}{7} = \frac{10}{21}$, which is a rational number.

(*ii*) Consider two rational numbers $-\frac{3}{5}$ and $\frac{1}{2}$.

Then their product $\frac{-3}{5}\times\frac{1}{2} = \left(\frac{-3\times1}{5\times2}\right) = \frac{-3}{10}$, which is a rational number.

(*iii*) Consider two rational numbers $\frac{-4}{3}$ and $\frac{-2}{5}$.

Then their product $\frac{-4}{3}\times\left(\frac{-2}{5}\right) = \frac{(-4)\times(-2)}{3\times5} = \frac{8}{15}$, which is a rational number.

2. Commutative Property

Two rational numbers can be multiplied in any order.

Thus, if $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers, then

$$\left(\frac{a}{b}\times\frac{c}{d}\right) = \left(\frac{c}{d}\times\frac{a}{b}\right).$$

Examples : (*i*) Let two rational numbers be $\frac{7}{8}$ and $\frac{2}{3}$. Then

$$\frac{7}{8}\times\frac{2}{3} = \frac{7\times2}{8\times3} = \frac{14}{24} = \frac{7}{12};\ \frac{2}{3}\times\frac{7}{8} = \frac{2\times7}{3\times8} = \frac{14}{24} = \frac{7}{12}$$

Hence, $\frac{7}{8}\times\frac{2}{3} = \frac{2}{3}\times\frac{7}{8}$

(*ii*) Let $\frac{-5}{2}$ and $\frac{1}{3}$ be two rational numbers. Then

$$\frac{-5}{2}\times\frac{1}{3} = \frac{-5\times1}{2\times3} = \frac{-5}{6};\ \frac{1}{3}\times\left(\frac{-5}{2}\right) = \frac{1\times(-5)}{3\times2} = \frac{-5}{6}$$

Hence $\left(\frac{-5}{2}\right)\times\frac{1}{3} = \frac{1}{3}\times\left(\frac{-5}{2}\right)$

3. Associative Property

While multiplying three or more rational numbers, they can be grouped in any order.

For any three rational numbers $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$,

$$\left(\frac{a}{b}\times\frac{c}{d}\right)\times\frac{e}{f} = \frac{a}{b}\times\left(\frac{c}{d}\times\frac{e}{f}\right)$$

Let the three rational numbers be $\frac{1}{2}, \frac{-3}{7}$ and $\frac{-5}{2}$. Then

$$\left[\frac{1}{2}\times\left(\frac{-3}{7}\right)\right]\times\left(\frac{-5}{2}\right) = \frac{-3}{14}\times\left(\frac{-5}{2}\right) = \frac{(-3)\times(-5)}{14\times2} = \frac{15}{28}$$

Also, $\frac{1}{2}\times\left[\frac{-3}{7}\times\left(\frac{-5}{2}\right)\right] = \frac{1}{2}\times\left[\frac{(-3)\times(-5)}{7\times2}\right] = \frac{1}{2}\times\frac{15}{14} = \frac{15}{28}$

Hence, $\left[\frac{1}{2}\times\left(\frac{-3}{7}\right)\right]\times\left(\frac{-5}{2}\right) = \frac{1}{2}\times\left[\frac{-3}{7}\times\left(\frac{-5}{2}\right)\right]$

4. Existence of Multiplicative Identity

For any rational number $\frac{a}{b}$, $\frac{a}{b}\times1 = 1\times\frac{a}{b} = \frac{a}{b}$.

1 is called the **multiplicative identity** for rational numbers.

Example : $\left(\frac{-3}{11}\right)\times1 = 1\times\left(\frac{-3}{11}\right) = \frac{-3}{11}$

5. Existence of Multiplicative Inverse

If $\frac{a}{b}$ is a rational number, then there exists a rational number $\frac{b}{a}$ such that $\frac{a}{b}\times\frac{b}{a} = 1$.

$\frac{a}{b}$ and $\frac{b}{a}$ are called **multiplicative inverse** or **reciprocal** of each other.

Examples : (*i*) The multiplicative inverse of $\frac{6}{7}$ is $\frac{7}{6}$.

$\because$ $\frac{6}{7}\times\frac{7}{6} = \frac{7}{6}\times\frac{6}{7} = 1$

(*ii*) The multiplicative inverse of $\frac{-2}{5}$ is $\frac{5}{-2}$.

$\because$ $\frac{-2}{5}\times\left(\frac{5}{-2}\right) = \frac{5}{-2}\times\left(\frac{-2}{5}\right) = 1$

6. Multiplicative Property of Zero

Every rational number when multiplied by zero gives zero.

For any rational number $\frac{a}{b}$, $\frac{a}{b}\times0 = 0\times\frac{a}{b} = 0$.

Examples : (*i*) $\frac{5}{13}\times0 = 0\times\frac{5}{13} = 0$ (*ii*) $\frac{-3}{11}\times0 = 0\times\left(\frac{-3}{11}\right) = 0$

7. Distributive Property

For any three rational numbers $\frac{a}{b}, \frac{c}{d}, \frac{e}{f}$, we have

$$\frac{a}{b}\times\left(\frac{c}{d}+\frac{e}{f}\right) = \left(\frac{a}{b}\times\frac{c}{d}\right)+\left(\frac{a}{b}\times\frac{e}{f}\right).$$

Example : Consider three rational numbers $\frac{2}{3}, \frac{4}{5}$ and $\frac{5}{7}$.

Then, $\frac{2}{3}\times\left(\frac{4}{5}+\frac{5}{7}\right) = \frac{2}{3}\times\left(\frac{28+25}{35}\right) = \frac{2}{3}\times\frac{53}{35} = \frac{106}{105}$

Also, $\left(\frac{2}{3}\times\frac{4}{5}\right)+\left(\frac{2}{3}\times\frac{5}{7}\right) = \frac{8}{15}+\frac{10}{21} = \frac{56+50}{105}=\frac{106}{105}$

Hence, $\frac{2}{3}\times\left(\frac{4}{5}+\frac{5}{7}\right) = \left(\frac{2}{3}\times\frac{4}{5}\right)+\left(\frac{2}{3}\times\frac{5}{7}\right)$

Example 20 : Write the reciprocal of each of the following rational numbers :

(*i*) 9 (*ii*) –13 (*iii*) $\frac{3}{7}$ (*iv*) $\frac{-5}{19}$ (*v*) $\frac{7}{-12}$

Solution : (*i*) The reciprocal of 9 is $\frac{1}{9}$.

(*ii*) The reciprocal of –13 is $-\frac{1}{13}$.

(*iii*) The reciprocal of $\frac{3}{7}$ is $\frac{1}{3/7}=\frac{7}{3}$.

(*iv*) The reciprocal of $\frac{-5}{19}$ is $\frac{1}{-5/19}=\frac{-19}{5}$.

(*v*) The reciprocal of $\frac{7}{-12}$ is $\frac{1}{7/(-12)}=\frac{-12}{7}$.

Example 21 : Find the reciprocal of the following :

(*i*) $\frac{2}{3}\times\frac{4}{7}$ (*ii*) $\frac{3}{7}\times\left(\frac{-5}{13}\right)$ (*iii*) $\frac{3}{-5}\times\frac{1}{3}$

Solution : (*i*) $\frac{2}{3}\times\frac{4}{7} = \frac{2\times4}{3\times7} = \frac{8}{21}$

The reciprocal of $\left(\frac{2}{3}\times\frac{4}{7}\right)$ *i.e.,* $\frac{8}{21}$ is $\frac{21}{8}$.

(*ii*) $\frac{3}{7}\times\left(\frac{-5}{13}\right) = \frac{3\times(-5)}{7\times13} = \frac{-15}{91}$

The reciprocal of $\frac{3}{7}\times\left(\frac{-5}{13}\right)$ *i.e.,* $\frac{-15}{91}$ is $\frac{-91}{15}$.

(*iii*) $\frac{3}{-5}\times\frac{1}{3} = \frac{3\times1}{-5\times3}=-\frac{1}{5}$

The reciprocal of $\frac{3}{-5}\times\frac{1}{3}$ *i.e.,* $-\frac{1}{5}$ is –5.

Laughter Time

Q. How many seconds are there in a year?
A. Twelve (January 2nd, February 2nd, March 2nd, April 2nd, ...)

Example 22 : Verify that :

(*i*) $\frac{7}{3}\times\left(\frac{9}{-5}\right)=\frac{9}{-5}\times\frac{7}{3}$ (*ii*) $\frac{-8}{9}\times\frac{13}{7}=\frac{13}{7}\times\left(\frac{-8}{9}\right)$

Solution : (*i*) $\frac{7}{3}\times\left(\frac{9}{-5}\right) = \frac{7\times9}{3\times(-5)}=\frac{21}{(-5)}=-\frac{21}{5}$

Also, $\frac{9}{-5}\times\frac{7}{3} = \frac{9\times7}{(-5)\times3}=\frac{21}{-5}=-\frac{21}{5}$

Hence, $\frac{7}{3}\times\left(\frac{9}{-5}\right) = \frac{9}{-5}\times\frac{7}{3}$

(*ii*) $\frac{-8}{9}\times\frac{13}{7} = \frac{-8\times13}{9\times7}=-\frac{104}{63}$

Also, $\frac{13}{7}\times\left(\frac{-8}{9}\right) = \frac{13\times(-8)}{7\times9}=-\frac{104}{63}$

Hence, $\frac{-8}{9}\times\frac{13}{7} = \frac{13}{7}\times\left(\frac{-8}{9}\right)$

Example 23 : Verify that :

(*i*) $\frac{12}{5}\times\left(\frac{-13}{24}\times\frac{35}{26}\right)=\left(\frac{12}{5}\times\frac{(-13)}{24}\right)\times\frac{35}{26}$ (*ii*) $\left(\frac{6}{7}\times\frac{(-7)}{10}\right)\times\frac{(-15)}{8}=\frac{6}{7}\times\left(\frac{-7}{10}\times\frac{(-15)}{8}\right)$

Solution : (*i*) $\frac{12}{5}\times\left(\frac{-13}{24}\times\frac{35}{26}\right) = \frac{12}{5}\times\left(\frac{-13\times35}{24\times26}\right)=\frac{12}{5}\times\left(\frac{-35}{48}\right) = \frac{-12\times35}{5\times48}=\frac{-7}{4}$

Also, $\left(\frac{12}{5}\times\frac{(-13)}{24}\right)\times\frac{35}{26} = \left(\frac{12\times(-13)}{5\times24}\right)\times\frac{35}{26}=\frac{-13}{10}\times\frac{35}{26} = \frac{-13\times35}{10\times26}=\frac{-7}{4}$

Hence, $\frac{12}{5}\times\left(\frac{-13}{24}\times\frac{35}{26}\right) = \left(\frac{12}{5}\times\frac{(-13)}{24}\right)\times\frac{35}{26}$

(*ii*) $\left(\frac{6}{7}\times\frac{(-7)}{10}\right)\times\frac{(-15)}{8} = \left(\frac{6\times(-7)}{7\times10}\right)\times\frac{(-15)}{8}=\frac{-3}{5}\times\frac{(-15)}{8} = \frac{(-3)\times(-15)}{5\times8}=\frac{9}{8}$

Also, $\frac{6}{7}\times\left(\frac{-7}{10}\times\frac{(-15)}{8}\right) = \frac{6}{7}\times\left(\frac{-7\times(-15)}{10\times8}\right) = \frac{6}{7}\times\frac{21}{16}=\frac{9}{8}$

Hence, $\left(\frac{6}{7}\times\frac{(-7)}{10}\right)\times\frac{(-15)}{8} = \frac{6}{7}\times\left(\frac{-7}{10}\times\frac{(-15)}{8}\right)$

Example 24 : Find each of the following products :

(*i*) $\frac{9}{11}\times\left(\frac{-66}{63}\right)$ (*ii*) $\frac{7}{6}\times\left(\frac{48}{-49}\right)$ (*iii*) $\frac{-8}{9}\times\left(\frac{27}{-64}\right)$ (*iv*) $\frac{-5}{19}\times\frac{57}{15}$

(*v*) $\frac{-15}{7}\times\left(\frac{-35}{60}\right)$ (*vi*) $\frac{-7}{-18}\times\left(\frac{27}{-14}\right)$

Solution : (*i*) $\frac{9}{11}\times\left(\frac{-66}{63}\right) = \frac{-9\times66}{11\times63}=-\frac{6}{7}$

(*ii*) $\frac{7}{6}\times\left(\frac{48}{-49}\right) = \frac{7\times48}{-6\times49}=-\frac{8}{7}$

(*iii*) $\frac{-8}{9}\times\left(\frac{27}{-64}\right) = \frac{-8\times27}{-9\times64}=\frac{3}{8}$

(*iv*) $\frac{-5}{19}\times\frac{57}{15} = \frac{-5\times57}{19\times15} = -1$

(*v*) $\frac{-15}{7}\times\left(\frac{-35}{60}\right) = \frac{15\times35}{7\times60}=\frac{5}{4}$

(*vi*) $\frac{-7}{-18}\times\left(\frac{27}{-14}\right) = \frac{-7\times27}{-18\times(-14)}=-\frac{3}{4}$

Example 25 : Simplify :

(*i*) $\left(\frac{7}{2}\times\frac{5}{3}\right)+\left(\frac{1}{6}\times\frac{3}{2}\right)-\left(\frac{12}{8}\times\frac{4}{3}\right)$ (*ii*) $\left(\frac{9}{-4}\times\frac{5}{3}\right)+\left(\frac{17}{2}\times\frac{5}{6}\right)$

(*iii*) $\left(\frac{8}{5}\times\frac{7}{3}\right)+\left(\frac{-15}{9}\times\frac{13}{2}\right)+\left(\frac{3}{2}\times\frac{1}{5}\right)$

Solution : (*i*) $\left(\frac{7}{2}\times\frac{5}{3}\right)+\left(\frac{1}{6}\times\frac{3}{2}\right)-\left(\frac{12}{8}\times\frac{4}{3}\right) = \frac{35}{6}+\frac{1}{4}-\frac{2}{1} = \frac{70+3-24}{12} = \frac{49}{12}$

(*ii*) $\left(\frac{9}{-4}\times\frac{5}{3}\right)+\left(\frac{17}{2}\times\frac{5}{6}\right) = \frac{-15}{4}+\frac{85}{12} = \frac{-45+85}{12} = \frac{40}{12}=\frac{10}{3}$

(iii) $\left(\frac{8}{5}\times\frac{7}{3}\right)+\left(\frac{-15}{9}\times\frac{13}{2}\right)+\left(\frac{3}{2}\times\frac{1}{5}\right) = \frac{56}{15}+\left(\frac{-65}{6}\right)+\frac{3}{10} = \frac{56}{15}-\frac{65}{6}+\frac{3}{10}$

$= \frac{112-325+9}{30} = -\frac{204}{30} = -\frac{34}{5}$

Example 26 : Verify : $x \times (y+z) = (x \times y) + (x \times z)$ if

(i) $x = \frac{3}{7},\ y = \frac{5}{6},\ z = \frac{5}{3}$ (ii) $x = \frac{5}{6},\ y = \frac{-8}{3},\ z = \frac{-13}{12}$

Solution : (i) $x = \frac{3}{7},\ y = \frac{5}{6},\ z = \frac{5}{3}$

L.H.S. $= x \times (y+z) = \frac{3}{7}\times\left(\frac{5}{6}+\frac{5}{3}\right)$

$= \frac{3}{7}\times\left(\frac{5+10}{6}\right) = \frac{3}{7}\times\frac{15}{6} = \frac{15}{14}$

R.H.S. $= (x \times y) + (x \times z) = \left(\frac{3}{7}\times\frac{5}{6}\right)+\left(\frac{3}{7}\times\frac{5}{3}\right) = \frac{15}{42}+\frac{15}{21} = \frac{15+30}{42} = \frac{45}{42} = \frac{15}{14}$

$\therefore$ L.H.S. = R.H.S.

Hence, $\frac{3}{7}\times\left(\frac{5}{6}+\frac{5}{3}\right) = \left(\frac{3}{7}\times\frac{5}{6}\right)+\left(\frac{3}{7}\times\frac{5}{3}\right)$.

(ii) $x = \frac{5}{6},\ y = \frac{-8}{3},\ z = \frac{-13}{12}$

L.H.S. $= x \times (y+z) = \frac{5}{6}\times\left[\frac{-8}{3}+\left(\frac{-13}{12}\right)\right]$

$= \frac{5}{6}\times\left(\frac{-8}{3}-\frac{13}{12}\right) = \frac{5}{6}\times\left(\frac{-32-13}{12}\right) = \frac{5}{6}\times\left(\frac{-45}{12}\right) = \frac{-25}{8}$

R.H.S. $= (x \times y) + (x \times z) = \left[\frac{5}{6}\times\left(\frac{-8}{3}\right)\right]+\left[\frac{5}{6}\times\left(\frac{-13}{12}\right)\right] = \frac{-40}{18}+\left(\frac{-65}{72}\right)$

$= \frac{-160-65}{72} = \frac{-225}{72} = \frac{-25}{8}$

$\therefore$ L.H.S. = R.H.S.

Hence, $\frac{5}{6}\times\left[\frac{-8}{3}+\left(\frac{-13}{12}\right)\right] = \left[\frac{5}{6}\times\left(\frac{-8}{3}\right)\right]+\left[\frac{5}{6}\times\left(\frac{-13}{12}\right)\right]$

Example 27 : Find the area of a rectangular field which is $21\frac{2}{3}$ m long and $9\frac{3}{5}$ m broad.

Solution : Length $= 21\frac{2}{3}$ m $= \frac{65}{3}$ m

Breadth $= 9\frac{3}{5}$ m $= \frac{48}{5}$ m

Area = length × breadth $= \frac{65}{3}\times\frac{48}{5}$ sq. m = 208 sq. m.

EXERCISE 1.4

Use Cordova Smart Class Software on the smart board in class to do Exercise.

1. Find each of the following products :

(i) $-\frac{3}{2}\times\frac{6}{7}$ (ii) $\frac{-12}{15}\times\left(\frac{20}{-3}\right)$ (iii) $\frac{17}{-5}\times(-10)$

(iv) $\frac{7}{26}\times\left(\frac{-52}{28}\right)$ (v) $\frac{-15}{13}\times\left(\frac{39}{-25}\right)$ (vi) $-8\times\left(\frac{-17}{24}\right)$

2. Verify each of the following :

(i) $\frac{2}{7}\times\left(\frac{-3}{8}\right)=\frac{-3}{8}\times\frac{2}{7}$ (ii) $-6\times\left(\frac{-26}{12}\right)=\frac{-26}{12}\times(-6)$ (iii) $\frac{-9}{7}\times\frac{13}{3}=\frac{13}{3}\times\left(\frac{-9}{7}\right)$

3. Find the multiplicative inverse (reciprocal) of the following :

(i) $\frac{3}{7}$ (ii) -9 (iii) $\frac{1}{-2}$ (iv) $-\frac{4}{5}$

4. Simplify :

(i) $-\frac{3}{5}\times\left(\frac{25}{12}+\frac{5}{4}\right)$ (ii) $\frac{2}{7}\times\left(\frac{7}{9}-\frac{35}{18}\right)$ (iii) $\frac{3}{5}\times\left(\frac{6}{9}-30\right)$

5. Verify :

(i) $\frac{2}{5}\times\left(\frac{4}{9}\times\frac{3}{1}\right)=\left(\frac{2}{5}\times\frac{4}{9}\right)\times\frac{3}{1}$ (ii) $\frac{-10}{9}\times\left(\frac{3}{-5}\times6\right)=\left[\frac{-10}{9}\times\left(\frac{3}{-5}\right)\right]\times6$

(iii) $\frac{-5}{7}\times\left(\frac{11}{3}\times\frac{14}{33}\right)=\left(\frac{-5}{7}\times\frac{11}{3}\right)\times\frac{14}{33}$

6. Find the reciprocal of :

(i) $\frac{5}{7}\times\left(\frac{-7}{16}\right)$ (ii) $\frac{-5}{4}\times\frac{1}{2}$ (iii) $\frac{-4}{9}\times\left(\frac{-3}{5}\right)$ (iv) $\frac{-3}{7}\times\frac{4}{9}$

7. Simplify :

(i) $\frac{8}{14}\times\frac{5}{4}\times\left(\frac{-49}{15}\right)+\frac{8}{5}\times\frac{15}{7}$ (ii) $\frac{15}{-13}\times\left(\frac{-7}{3}\right)+(-5)\times\frac{4}{13}$ (iii) $\frac{4}{99}\times\frac{9}{5}-\frac{3}{5}\times\frac{4}{99}$

8. Use the distributive property to evaluate :

(i) $\frac{9}{13}\times3\frac{1}{5}-2\frac{1}{3}\times\frac{9}{13}$ (ii) $6\frac{2}{5}\times\frac{3}{7}+\frac{4}{7}\times6\frac{2}{5}$ (iii) $6\frac{2}{3}\times\frac{3}{2}+\frac{5}{2}\times6\frac{2}{3}+\frac{7}{2}\times6\frac{2}{3}$

9. Verify :

(i) $\frac{-4}{3}\times\left(\frac{6}{-5}\times\frac{8}{9}\right)=\left[\frac{4}{-3}\times\left(\frac{6}{-5}\right)\right]\times\frac{8}{9}$ (ii) $\frac{-9}{5}\times\left[\frac{7}{9}\times\left(\frac{2}{-3}\right)\right]=\left(-\frac{9}{5}\times\frac{7}{9}\right)\times\left(\frac{2}{-3}\right)$

(iii) $-\frac{5}{9}\times\left[\frac{3}{26}+\left(\frac{-2}{13}\right)\right]=\left(-\frac{5}{9}\times\frac{3}{26}\right)+\left[\frac{-5}{9}\times\left(\frac{-2}{13}\right)\right]$ (iv) $\frac{6}{13}\times\left(\frac{-2}{5}\right)+\frac{1}{5}\times\frac{6}{13}=\frac{6}{13}\left(-\frac{2}{5}+\frac{1}{5}\right)$

10. Find the area of a square field whose side is $7\frac{1}{4}$ m.

DIVISION OF RATIONAL NUMBERS

Division is the inverse of multiplication.

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers such that $\frac{c}{d} \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$.

Dividend : The number to be divided is called the dividend *e.g.,* $\frac{a}{b}$ is the dividend.

Divisor : The number which divides the dividend is called the divisor *e.g.,* $\frac{c}{d}$ is the divisor.

Quotient : When dividend is divided by the divisor, the result obtained is called the quotient. *e.g.,* $\frac{a}{b} \times \frac{d}{c}$ is the quotient.

Note : *Division by zero is not defined.*

Properties of Division of Rational Numbers

1. Closure Property

If $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers where $\frac{c}{d} \neq 0$, then $\frac{a}{b} \div \frac{c}{d}$ is also a rational number.

Example : Consider two rational numbers $\frac{2}{3}$ and $\frac{5}{7}$. Then $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$, which is a rational number.

2. Division by 1

For every rational number $\frac{a}{b}$, $\frac{a}{b} \div 1 = \frac{a}{b}$.

For example : $\frac{3}{4} \div 1 = \frac{3}{4} \times \frac{1}{1} = \frac{3}{4}$

Similarly, $\frac{a}{b} \div (-1) = -\frac{a}{b}$

Example 28 : Divide :

(i) $\frac{3}{7} \div \left(\frac{-5}{4}\right)$ (ii) $\frac{-7}{3} \div \frac{5}{6}$ (iii) $\frac{-11}{56} \div \left(\frac{44}{-7}\right)$ (iv) $\frac{-5}{13} \div \frac{45}{78}$

Solution : (i) $\frac{3}{7} \div \left(\frac{-5}{4}\right) = \frac{3}{7} \times \left(\frac{-4}{5}\right) = \frac{-12}{35}$

(ii) $\frac{-7}{3} \div \frac{5}{6} = \frac{-7}{3} \times \frac{6}{5} = \frac{-7 \times 6}{3 \times 5} = \frac{-14}{5}$

(iii) $\frac{-11}{56} \div \left(\frac{44}{-7}\right) = \frac{-11}{56} \times \left(\frac{-7}{44}\right) = \frac{11 \times 7}{56 \times 44} = \frac{1}{32}$

(iv) $\frac{-5}{13} \div \frac{45}{78} = \frac{-5}{13} \times \frac{78}{45} = \frac{-5 \times 78}{13 \times 45} = -\frac{2}{3}$

Example 29 : The product of two rational numbers is $\frac{-36}{5}$. If one number is $\frac{9}{16}$, what is the other number?

Solution : Let the other number be x.

Then, $\frac{9}{16} \times x = \frac{-36}{5}$

$\therefore \quad x = \frac{-36}{5} \div \frac{9}{16} = \frac{-36}{5} \times \frac{16}{9} = \frac{-64}{5}$

The other number is $\frac{-64}{5}$.

Example 30 : By what number should we multiply $\frac{-5}{14}$ to get $\frac{-1}{6}$?

Solution : Let x be the required number.

Then, $\frac{-5}{14} \times x = \frac{-1}{6}$

$\Rightarrow \quad x = \frac{-1}{6} \div \left(\frac{-5}{14}\right)$

$\Rightarrow \quad x = \frac{(-1)}{6} \times \left(\frac{-14}{5}\right) = \frac{7}{15}$

The required number is $\frac{7}{15}$.

Example 31 : Divide the sum of $\frac{2}{7}$ and $\frac{3}{5}$ by their product.

Solution : Sum of $\frac{2}{7}$ and $\frac{3}{5} = \frac{2}{7} + \frac{3}{5} = \frac{10+21}{35} = \frac{31}{35}$

Product of $\frac{2}{7}$ and $\frac{3}{5} = \frac{2}{7} \times \frac{3}{5} = \frac{2 \times 3}{7 \times 5} = \frac{6}{35}$

Dividing the sum by product, we get

$$\frac{31}{35} \div \frac{6}{35} = \frac{31}{35} \times \frac{35}{6} = \frac{31}{6}$$

Example 32 : Divide the sum of $\frac{-9}{7}$ and $\frac{7}{3}$ by the difference of $\frac{3}{5}$ and $\frac{2}{7}$.

Solution : Sum of $\frac{-9}{7}$ and $\frac{7}{3} = \frac{-9}{7} + \frac{7}{3} = \frac{-27+49}{21} = \frac{22}{21}$

Difference of $\frac{3}{5}$ and $\frac{2}{7} = \frac{3}{5} - \frac{2}{7} = \frac{21-10}{35} = \frac{11}{35}$

Dividing the sum by the difference, we get

$$\frac{22}{21} \div \frac{11}{35} = \frac{22}{21} \times \frac{35}{11} = \frac{10}{3}$$

Example 33 : By what number should we divide $\frac{-63}{15}$ to get -3?

Solution : Let the required number be x.

Then, $\frac{-63}{15} \div x = -3$

$\Rightarrow \quad \frac{-63}{15} \times \frac{1}{x} = -3$

$\Rightarrow \quad \frac{1}{x} = -3 \div \left(\frac{-63}{15}\right)$

$\Rightarrow \quad \frac{1}{x} = -3 \times \left(\frac{15}{-63}\right)$

$\Rightarrow \quad \frac{1}{x} = \frac{3 \times 15}{63} = \frac{5}{7}$

$\Rightarrow \quad x = \frac{7}{5}$

The required number is $\frac{7}{5}$.

Example 34 : The cost of $2\frac{3}{5}$ m of cloth is ₹65. Find the cost of cloth per metre.

Solution : Cost of cloth per metre $= ₹\left(65 \div 2\frac{3}{5}\right) = ₹\left(65 \div \frac{13}{5}\right) = ₹65 \times \frac{5}{13} = ₹25$

Example 35 : If $a = \frac{2}{5}$ and $b = \frac{1}{3}$, then find $(a + b) \div (a - b)$.

Solution :

$$a + b = \frac{2}{5}+\frac{1}{3} = \frac{6+5}{15} = \frac{11}{15}$$

$$a - b = \frac{2}{5}-\frac{1}{3} = \frac{6-5}{15} = \frac{1}{15}$$

$$\therefore \quad (a + b) \div (a - b) = \frac{11}{15} \div \frac{1}{15} = \frac{11}{15}\times\frac{15}{1} = 11$$

EXERCISE 1.5

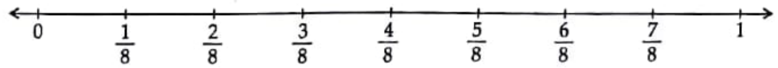
Use **Cordova Smart Class Software** on the **smart board** in class to do **Exercise.**

1. Divide :
   (i) $\frac{5}{13} \div \frac{10}{39}$ (ii) $\frac{3}{-15} \div \frac{7}{5}$ (iii) $8\frac{2}{3} \div \frac{13}{3}$
   (iv) $\frac{25}{36} \div \frac{15}{12}$ (v) $\frac{36}{44} \div \frac{4}{11}$ (vi) $\frac{35}{63} \div \frac{7}{21}$
2. By what number should $\frac{-4}{35}$ be multiplied to get $\frac{8}{105}$?
3. By what number should $\frac{5}{7}$ be divided to get $\frac{-15}{56}$?
4. If $x = \frac{5}{4}$ and $y = -\frac{1}{3}$, find the value of $(x + y) \div (x - y)$.
5. Divide the sum of $\frac{3}{11}$ and $\frac{2}{5}$ by their product.
6. The product of two numbers is $\frac{92}{3}$. If one of the numbers is $\frac{56}{3}$, what is the other number?
7. Simplify :
   (i) $\frac{4}{7} \div \left(\frac{2}{9} \div \frac{14}{27}\right)$ (ii) $\left(-\frac{5}{39} \div \frac{20}{13}\right) \div \left(\frac{-5}{3}\right)$ (iii) $\left(\frac{1}{5} \div 3\right) \div \frac{1}{6}$ (iv) $\left(-\frac{3}{26} \div \frac{6}{33}\right) \div \frac{11}{13}$
8. By what number should we divide $\frac{4}{5}$ to get $\frac{8}{125}$?
9. Divide the sum of $\frac{78}{12}$ and $\frac{8}{3}$ by their difference.
10. Simplify :
    (i) $-\frac{3}{5} \div \left(-\frac{12}{35} \div \frac{1}{28}\right)$ (ii) $\left(-\frac{2}{3} \div \frac{9}{7}\right) \div \left(\frac{5}{-9}\right)$
11. The product of two numbers is $\frac{28}{121}$. If one of the numbers is $\frac{2}{3}$, find the other number.
12. By what number should $\frac{44}{-7}$ be divided to get $\frac{-11}{3}$?

RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

Let us find rational numbers between two given rational numbers.

How many rational numbers will be there between $\frac{3}{8}$ and $\frac{7}{8}$. At a glance we may say that the rational numbers are $\frac{4}{8}, \frac{5}{8}$ and $\frac{6}{8}$, but besides these, there are many more rational numbers between them.



There are rational numbers between $\frac{3}{8}$ and $\frac{4}{8}$, between $\frac{4}{8}$ and $\frac{5}{8}$, between $\frac{5}{8}$ and $\frac{6}{8}$ and also between $\frac{6}{8}$ and $\frac{7}{8}$. We may write $\frac{3}{8}$ and $\frac{7}{8}$ as $\frac{30}{80}$ and $\frac{70}{80}$ respectively.

Now the numbers $\frac{31}{80}, \frac{32}{80}, \frac{33}{80}, \frac{34}{80}, \ldots \frac{69}{80}$ are all between $\frac{30}{80}$ and $\frac{70}{80}$.

The same numbers may be written as $\frac{300}{800}$ and $\frac{700}{800}$. Then the numbers $\frac{301}{800}, \frac{302}{800}, \ldots \frac{699}{800}$ will be between $\frac{300}{800}$ and $\frac{700}{800}$.

So, there are infinite rational numbers between two rational numbers.

We can also use the idea of mean to find rational numbers between two given rational numbers.

If a and b are two rational numbers such that $a < b$, then $a < \frac{a+b}{2} < b$.

Example 36 : Find three rational numbers between –6 and –5.

Solution : The rational number between –6 and –5 $= \frac{-6-5}{2} = \frac{-11}{2}$

The rational number between –6 and $\frac{-11}{2} = \left(-6-\frac{11}{2}\right)\times\frac{1}{2} = \frac{-23}{4}$

The rational number between $\frac{-23}{4}$ and –5 is $\left(\frac{-23}{4}-\frac{5}{1}\right)\times\frac{1}{2} = \frac{-43}{8}$

Hence, the required three rational numbers are $-\frac{11}{2}, \frac{-23}{4}$ and $-\frac{43}{8}$.

Example 37 : Find five rational numbers between $\frac{-5}{6}$ and $\frac{3}{8}$.

Solution : The given numbers are $\frac{-5}{6}$ and $\frac{3}{8}$.

L.C.M. of denominators 6 and 8 is 24.

$$\therefore \quad -\frac{5}{6} = -\frac{5\times4}{6\times4} = \frac{-20}{24}\ ;\ \frac{3}{8} = \frac{3\times3}{8\times3} = \frac{9}{24}$$

We can now write the rational numbers between $\frac{-20}{24}$ and $\frac{9}{24}$ as

$\frac{-19}{24}, \frac{-18}{24}, \frac{-17}{24}, \frac{-16}{24}, \ldots, \frac{6}{24}, \frac{7}{24}, \frac{8}{24}$.

We may choose any five of them.

EXERCISE 1.6

Use Cordova Smart Class Software on the smart board in class to do Exercise.

1. Find three rational numbers between $\frac{-3}{7}$ and $\frac{-2}{7}$.
2. Find four rational numbers between $\frac{-3}{5}$ and $\frac{-2}{5}$.
3. Find three rational numbers between 0 and 1.
4. Find three rational numbers between

   (i) $\frac{2}{3}$ and 3 (ii) $-\frac{1}{3}$ and $\frac{1}{2}$ (iii) $\frac{1}{6}$ and $\frac{5}{7}$ (iv) $-\frac{1}{9}$ and $\frac{2}{9}$

FACTS TO REMEMBER

- Any number which can be expressed in the form p/q, where p and q are integers and $q \neq 0$ is called a rational number.
- The integer zero is a rational number.
- A rational number is said to be in its lowest form if its numerator and denominator have no common divisor other than 1.
- Every fraction as well as every integer is a rational number.
- Between two rational numbers, there exist infinite rational numbers.
- If x and y are two rational numbers, then $\frac{x+y}{2}$ is a rational number between them.
- If x and y are two rational numbers, then their sum, product, difference and quotient $\frac{x}{y}$ $(y \neq 0)$, will also be rational numbers.
- If x and y are two rational numbers, then
  (i) $x + y = y + x$ (ii) $x \times y = y \times x$
- If x, y and z are three rational numbers, then
  (i) $x + (y + z) = (x + y) + z$ (ii) $x \times (y + z) = (x \times y) + (x \times z)$ (iii) $x \times (y \times z) = (x \times y) \times z$
- 1 is the multiplicative identity and 0 is the additive identity for rational numbers.
- The additive inverse of a rational number $\frac{a}{b}$ is $-\frac{a}{b}$ and multiplicative inverse of $\frac{a}{b}$ is $\frac{b}{a}$ such that $\frac{a}{b} \times \frac{b}{a} = 1$.

HOTS QUESTIONS

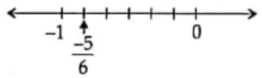
1. If $\frac{1}{5}$ of a number is subtracted from $\frac{1}{2}$ of that number, the difference is 5 less than $\frac{1}{3}$ of the same number. Find the number.
2. A student was asked to divide a number by $\frac{5}{4}$. By mistake he divided the given number by $\frac{4}{5}$. His answer exceeds the correct answer by 54. Find the number.
3. Find the sum of additive inverse and multiplicative inverse of 5.

ANSWERS

Exercise 1.1

1. $\frac{0}{5}, 2, \frac{3}{4}$
2. Numerator : –2, 4, 0, 5, 3. Denominator 3, 1, 3, 1, –1
3. (i) $\frac{5}{-3}$ (ii) $\frac{35}{-21}$ (iii) $\frac{-20}{12}$ (iv) $\frac{15}{-9}$
4. (i) $\frac{9}{12}$ (ii) $\frac{15}{20}$ (iii) $\frac{-12}{-16}$ (iv) $\frac{21}{28}$
5. (i) $\frac{3}{13}$ (ii) $\frac{-3}{7}$ (iii) $\frac{1}{6}$ (iv) $\frac{-7}{5}$
6. (i) $\frac{3}{5}$ (ii) $\frac{4}{7}$ (iii) $\frac{8}{9}$ (iv) $\frac{6}{11}$
7. (i) $-\frac{4}{6}, \frac{-6}{9}, \frac{-8}{12}$ (ii) $\frac{6}{10}, \frac{9}{15}, \frac{12}{20}$ (iii) $\frac{14}{-12}, \frac{21}{-18}, \frac{28}{-24}$

Exercise 1.2

1. (i) $\frac{3}{4}$ (ii) $-\frac{1}{2}$ (iii) $\frac{8}{15}$ (iv) $-\frac{1}{2}$
2. (i)


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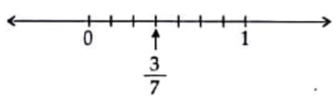

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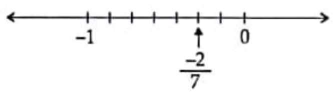

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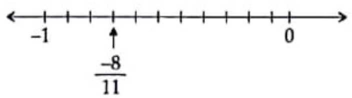


3. (i) $\frac{-13}{20}, \frac{17}{-30}, \frac{8}{-15}, \frac{-3}{10}$ (ii) $\frac{-13}{5}, \frac{7}{-3}, -2, \frac{2}{3}$
4. (i) $\frac{3}{-8}, \frac{-5}{12}, \frac{-7}{6}, \frac{-11}{7}$ (ii) $\frac{-11}{9}, \frac{7}{-5}, \frac{-17}{11}, \frac{13}{-8}$
5. (i) > (ii) < (iii) < (iv) = (v) < (vi) >
6. $-\frac{3}{2}, -\frac{5}{2}, -\frac{7}{2}, -2, -\frac{9}{2}$ 7. $-\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}$

Exercise 1.3

1. (i) $\frac{13}{40}$ (ii) $-\frac{23}{30}$ (iii) $\frac{29}{6}$ (iv) $\frac{11}{21}$
2. (i) $-\frac{17}{21}$ (ii) $-\frac{50}{91}$ (iii) $-\frac{37}{18}$ (iv) $\frac{11}{10}$
3. (i) $\frac{25}{24}$ (ii) $-\frac{69}{144}$ (iii) $\frac{8}{15}$ (iv) $-\frac{16}{3}$
4. Commutative law of addition on Rational numbers.
5. Associative law of addition on Rational numbers.
6. (i) $\frac{7}{30}$ (ii) $-\frac{17}{24}$
7. (i) $\frac{62}{63}$ (ii) $-\frac{13}{24}$ (iii) $-\frac{29}{15}$
8. (i) $\frac{3}{7}$ (ii) $\frac{16}{3}$ (iii) $-\frac{7}{9}$ (iv) $-\frac{11}{5}$
9. $\frac{7}{3}$ 10. $-\frac{28}{9}$ 11. $\frac{1}{4}$ 12. $-\frac{16}{15}$
13. $\frac{2}{3}$ 14. (i) $\frac{2}{3}$ (ii) $-\frac{5}{3}$ 15. $-\frac{1}{10}$

Exercise 1.4

1. (i) $-\frac{9}{7}$ (ii) $\frac{16}{3}$ (iii) 34 (iv) $-\frac{1}{2}$ (v) $\frac{9}{5}$ (vi) $\frac{17}{3}$
3. (i) $\frac{7}{3}$ (ii) $-\frac{1}{9}$ (iii) –2 (iv) $-\frac{5}{4}$
4. (i) –2 (ii) $-\frac{1}{3}$ (iii) $-\frac{88}{5}$
6. (i) $-\frac{16}{5}$ (ii) $-\frac{8}{5}$ (iii) $\frac{15}{4}$ (iv) $-\frac{21}{4}$
7. (i) $\frac{23}{21}$ (ii) $\frac{15}{13}$ (iii) $\frac{8}{165}$
8. (i) $\frac{3}{5}$ (ii) $\frac{32}{5}$ (iii) 50 10. $52\frac{9}{16}$ sq. m

Exercise 1.5

1. (i) $\frac{3}{2}$ (ii) $-\frac{1}{7}$ (iii) 2 (iv) $\frac{5}{9}$ (v) $\frac{9}{4}$ (vi) $\frac{5}{3}$

2. $-\frac{2}{3}$ 3. $-\frac{8}{3}$ 4. $\frac{11}{19}$ 5. $\frac{37}{6}$

6. $\frac{23}{14}$ 7. (i) $\frac{4}{3}$ (ii) $\frac{1}{20}$ (iii) $\frac{2}{5}$ (iv) $-\frac{3}{4}$

8. $\frac{25}{2}$ 9. $\frac{55}{23}$ 10. (i) $\frac{1}{16}$ (ii) $\frac{14}{15}$

11. $\frac{42}{121}$ 12. $\frac{12}{7}$

Exercise 1.6

1. $\frac{-5}{14}, \frac{-9}{28}, \frac{-17}{56}$ 2. $-\frac{1}{2}, \frac{-11}{20}, \frac{-19}{40}, \frac{-41}{80}$ 3. $\frac{1}{2}, \frac{1}{4}, \frac{3}{4}$

4. (i) $\frac{11}{6}, \frac{15}{12}, \frac{29}{12}$ (ii) $-\frac{1}{8}, \frac{1}{12}, \frac{7}{24}$ (iii) $\frac{51}{168}, \frac{37}{84}, \frac{97}{168}$

(iv) $-\frac{1}{36}, \frac{1}{18}, \frac{5}{36}$

HOTS

1. 150 2. 120 3. $-4\frac{4}{5}$

NCERT Exemplar Problems

Example 1 : $\frac{2}{5}$ of total number of students of a school come by car while $\frac{1}{4}$ of students come by bus to school. All the other students walk to school of which $\frac{1}{3}$ walk on their own and the rest are escorted by their parents. If 224 students come to school walking on their own, how many students study in that school?

Solution : Students walking to school on their own = 224 [Given]

Let the total number of students in the school be x.

Number of students walk to school $= x - \left(\frac{2}{5} \text{ of } x + \frac{1}{4} \text{ of } x\right)$

$= x - \left(\frac{2x}{5} + \frac{x}{4}\right)$

$= x - \left(\frac{8x+5x}{20}\right) = x - \frac{13}{20}x = \frac{7x}{20}$

Students walking to school on their own $= \frac{1}{3}$ of $\frac{7x}{20}$

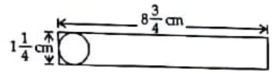
$\Rightarrow \quad 224 = \frac{1}{3} \times \frac{7x}{20} \Rightarrow x = \frac{224 \times 20 \times 3}{7}$

$\Rightarrow \quad x = 32 \times 20 \times 3 = 1920$

Hence, total number of students in the school is 1920.

Example 2 : Shalini has to cut out circles of diameter $1\frac{1}{4}$ cm from an aluminium strip of dimensions $8\frac{3}{4}$ cm by $1\frac{1}{4}$ cm. How many full circles can Shalini cut? Also calculate the wastage of the Aluminium strip.

Solution : Given: Diameter of circle $= 1\frac{1}{4}$ cm $= \frac{5}{4}$ cm



Length of aluminium strip $= 8\frac{3}{4}$ cm $= \frac{35}{4}$ cm

Width of aluminium strip $= 1\frac{1}{4}$ cm $= \frac{5}{4}$ cm

Area of circle $= \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \pi\frac{d^2}{4}$ [d = Diameter]

$= \frac{\pi}{4} \times \left(\frac{5}{4}\right)^2$ cm^2

$= \frac{\pi}{4} \times \frac{25}{16}$ cm^2

$= \frac{22}{7} \times \frac{1}{4} \times \frac{25}{16}$ cm^2 ...(i)

Area of strip $= \frac{35}{4} \times \frac{5}{4} = \frac{35 \times 5}{16} = \frac{175}{16}$ cm^2

No. of circles which can be cut out from aluminium strip

$= \frac{\text{Length of strip}}{\text{Diameter of circle}}$

$= \frac{35/4}{5/4} = \frac{35}{4} \times \frac{4}{5} = 7$

Total area required for seven circles $= 7 \times \frac{22}{7 \times 4} \times \frac{25}{16} \text{cm}^2$ [Using (i)]

$= \frac{11 \times 25}{32} \text{cm}^2 = \frac{275}{32} \text{cm}^2$

Wastage from aluminium strip = Area of strip − Area of 7 circles

$= \left(\frac{175}{16} - \frac{275}{32}\right) \text{cm}^2$

$= \frac{350 - 275}{32} \text{cm}^2 = \frac{75}{32} \text{cm}^2$

Hence, 7 circles can be cut out from aluminium strip and wastage is $\frac{75}{32} \text{cm}^2$.

Example 3 : The table shows the portion of some common materials that are recycled.

Material	Recycled
Paper	$\frac{5}{11}$
Aluminium cans	$\frac{5}{8}$
Glass	$\frac{2}{5}$
Scrap	$\frac{3}{4}$

(*a*) Is the rational number expressing the amount of paper recycled more than $\frac{1}{2}$ or less than $\frac{1}{2}$?

(*b*) Which items have a recycled amount less than $\frac{1}{2}$?

(*c*) Is the quantity of aluminium cans recycled more (or less) than half of the quantity of aluminium cans?

(*d*) Arrange the rate of recycling the materials from the greatest to the smallest.

Solution : (*a*) $\frac{5}{11}$ is less than half i.e., $\frac{5}{11} < \frac{1}{2}$

(*b*) Paper and glass have recycling amount less than $\frac{1}{2}$.

(*c*) More than half

(*d*) Paper $= \frac{5}{11} = 0.454$, Aluminium cans $= \frac{5}{8} = 0.625$, Glass $= \frac{2}{5} = 0.4$, Scrap $= \frac{3}{4} = 0.75$

Hence, Scrap > Aluminium cans > Paper > Glass

2 Exponents

Use Cordova Smart Class Software on the smart board in class to learn about exponents.

EXPONENTS

We have already learnt that 2 × 2 × 2 × 2 can be written in the exponential form as 2^4, where 2 is the base and 4 is the exponent. It is read as "two raised to the power 4."

Thus, if x is a rational number and n is a positive integer, then $x^n = x \times x \times x \times x \ldots n$ times, where x is the **base** and n is called **exponent** or **power.**

Negative Integral Exponent of a Rational Number

For any non-zero rational number a, $a^{-n} = \frac{1}{a^n}$ [where n is a positive integer]

Which implies that a^{-n} is the reciprocal of a^n.

Example 1 : Express each of the following as a rational number in the form $\frac{p}{q}$:

(*i*) 2^{-4} (*ii*) $(-3)^{-4}$ (*iii*) $\frac{1}{3^{-3}}$

Solution : We know that, $a^{-n} = \frac{1}{a^n}$, where a is a non-zero rational number and n is a positive integer.

(*i*) $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$ $\left[\because a^{-n} = \frac{1}{a^n}\right]$

(*ii*) $(-3)^{-4} = \frac{1}{(-3)^4} = \frac{1}{(-3)\times(-3)\times(-3)\times(-3)} = \frac{1}{81}$ $\left[\because a^{-n} = \frac{1}{a^n}\right]$

(*iii*) $\frac{1}{3^{-3}} = 3^3 = 27$ $\left[\because \frac{1}{a^{-n}} = a^n\right]$

Example 2 : Express each of the following as a rational number in the form $\frac{p}{q}$:

(*i*) $\left(\frac{3}{4}\right)^{-2}$ (*ii*) $\left(-\frac{2}{5}\right)^{-3}$

Solution : (*i*) $\left(\frac{3}{4}\right)^{-2} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$ $\left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$

(*ii*) $\left(-\frac{2}{5}\right)^{-3} = \left(-\frac{5}{2}\right)^3 = \frac{-125}{8}$ $\left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$

Example 3 : Write the multiplicative inverse of each of the following :

(i) -23 (ii) $\frac{1}{6}$ (iii) $9^2 \div \frac{1}{5^{-2}}$ (iv) $3^2 \times \frac{1}{2^3}$

Solution : (i) The multiplicative inverse of -23 is $(-23)^{-1} = \frac{1}{(-23)^1} = -\frac{1}{23}$

(ii) The multiplicative inverse of $\frac{1}{6} = \left(\frac{1}{6}\right)^{-1} = \frac{1}{\frac{1}{6}} = 6$

(iii) $9^2 \div \frac{1}{5^{-2}} = 81 \div 5^2$ $\left[\because \frac{1}{a^{-n}} = a^n\right]$

$= 81 \times \frac{1}{25} = \frac{81}{25}$

The multiplicative inverse of $9^2 \div \frac{1}{5^{-2}}$ i.e., $\frac{81}{25} = \left(\frac{81}{25}\right)^{-1} = \frac{25}{81}$

(iv) $3^2 \times \frac{1}{2^3} = 9 \times \frac{1}{8} = \frac{9}{8}$

The multiplicative inverse of $3^2 \times \frac{1}{2^3}$ i.e., $\frac{9}{8}$ is $\left(\frac{9}{8}\right)^{-1} = \frac{1}{\left(\frac{9}{8}\right)} = \frac{8}{9}$

Let $\frac{a}{b}$ be any rational number and n be a positive integer, then $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$.

Also, we define $a^0 = 1$

The multiplicative inverse of a is a^{-1}.

In general, multiplicative inverse of a^n is a^{-n}, for any positive integer n.

Example 4 : Express each of the following with positive exponent :

(i) $\left(\frac{3}{5}\right)^{-3}$ (ii) 7^{-5} (iii) $\left(-\frac{2}{3}\right)^{-6}$

Solution : (i) $\left(\frac{3}{5}\right)^{-3} = \frac{1}{\left(\frac{3}{5}\right)^3} = \frac{1}{\frac{3^3}{5^3}} = \frac{5^3}{3^3} = \left(\frac{5}{3}\right)^3$

(ii) $7^{-5} = \frac{1}{7^5}$ $\left[\because a^{-n} = \frac{1}{a^n}\right]$

(iii) $\left(\frac{-2}{3}\right)^{-6} = \frac{1}{\left(\frac{-2}{3}\right)^6} = \frac{1}{(-2)^6/3^6} = \left(\frac{-3}{2}\right)^6$ $\left[\because a^{-n} = \frac{1}{a^n}\right]$

Example 5 : Simplify :

(i) $7^{-2} + 3^{-1}$ (ii) $(5^{-1} - 3^{-1})^{-1}$ (iii) $(6^{-1} + 3^{-2}) \times \left(\frac{5}{2}\right)^{-1}$

Solution : (i) $7^{-2} + 3^{-1} = \frac{1}{7^2} + \frac{1}{3} = \frac{1}{49} + \frac{1}{3} = \frac{1}{49} \times 3 = \frac{3}{49}$

(ii) $(5^{-1} - 3^{-1})^{-1} = \left(\frac{1}{5} - \frac{1}{3}\right)^{-1} = \left(\frac{3-5}{15}\right)^{-1} = \left(-\frac{2}{15}\right)^{-1}$

$= -\frac{15}{2}$ $\left[\because \left(\frac{a}{b}\right)^{-1} = \left(\frac{b}{a}\right)\right]$

(iii) $(6^{-1} + 3^{-2}) \times \left(\frac{5}{2}\right)^{-1} = \left(\frac{1}{6} + \frac{1}{3^2}\right) \times \left(\frac{2}{5}\right)$

$= \left(\frac{1}{6} + \frac{1}{9}\right) \times \frac{2}{5} = \left(\frac{3+2}{18}\right) \times \frac{2}{5} = \frac{5}{18} \times \frac{2}{5} = \frac{1}{9}$

Example 6 : By what number should $(-28)^{-1}$ be divided so that the quotient may be 4^{-1} ?

Solution : Let x be the required number.

Then, $(-28)^{-1} \div x = 4^{-1}$

$\Rightarrow \left(\frac{-1}{28}\right) \times \frac{1}{x} = \frac{1}{4}$

$\Rightarrow \frac{1}{x} = \frac{1}{4} \div \left(\frac{-1}{28}\right) = \frac{1}{4} \times (-28) = -7$

$\Rightarrow x = -\frac{1}{7}$

Hence, $(-28)^{-1}$ should be divided by $-\frac{1}{7}$ to get 4^{-1}.

EXERCISE 2.1

Use Cordova Smart Class Software on the smart board in class to do Exercise.

1. Express each of the following as a rational number :

(i) $(-2)^5$ (ii) $\left(-\frac{5}{6}\right)^3$ (iii) $(2)^{-4}$ (iv) $(-4)^{-4}$ (v) $\left(-\frac{4}{5}\right)^{-2}$

2. Write the multiplicative inverse of the following :

(i) $(7)^{-2}$ (ii) $(-4)^3 \times \frac{1}{2^3}$ (iii) $5^{-2} + 5^{-4}$ (iv) $\left(\frac{3}{4}\right)^{-2}$

3. Show that $\left[\left(\frac{3}{5}\right)^{-1} - \left(\frac{1}{3}\right)^{-1}\right]^{-1} = -\frac{3}{4}$.
4. By what number should $(-36)^{-1}$ be divided so that the quotient is 9^{-1} ?
5. By what number should $(-6)^{-1}$ be multiplied so that the product is 27^{-1} ?
6. Simplify :

(i) $(3^{-1} \times 3^{-2}) \div 3^{-3}$ (ii) $(4^2 - 3^2) \times \left(\frac{7}{2}\right)^{-2}$ (iii) $(5^{-1} \times 6^{-1}) \div 10^{-1}$ (iv) $(6^{-1} + 7^{-1})^2$

7. If $a = -2$, $b = 3$, evaluate each of the following :

(i) $25a^{-3}$ (ii) $27a^2b^{-3}$ (iii) $162(-a)^{-1}(b)^{-3}$

LAWS OF EXPONENTS WITH INTEGRAL POWERS

In the previous class, we have learnt the laws of exponents of rational numbers when exponents are whole numbers. Let us revise them :

(i) $a^m \times a^n = a^{m+n}$ (ii) $\frac{a^m}{a^n} = a^{m-n}$, $m > n$ (iii) $(a^m)^n = a^{mn}$

(iv) $(ab)^n = a^n b^n$ (v) $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

These laws also hold good for negative integral exponents.

Law I : If x is a non-zero rational number and m and n are any two integers, then $x^m \times x^n = x^{m+n}$.

Example 7 : Show that :

(i) $3^7 \times 3^{-5} = 3^{7+(-5)}$ (ii) $\left(\frac{3}{5}\right)^{-4} \times \left(\frac{3}{5}\right)^{-3} = \left(\frac{3}{5}\right)^{-4+(-3)}$ (iii) $(-2)^5 \times (-2)^{-8} = (-2)^{5+(-8)}$

Solution : (i)

$$\text{L.H.S.} = 3^7 \times 3^{-5} = 3^7 \times \frac{1}{3^5} \qquad \left[\because a^{-n} = \frac{1}{a^n}\right]$$

$$= \frac{3^7}{3^5} = \frac{3\times3\times3\times3\times3\times3\times3}{3\times3\times3\times3\times3} = 3 \times 3 = 3^2$$

$$\text{R.H.S.} = 3^{7+(-5)} = 3^{7-5} = 3^2$$

Hence, L.H.S. = R.H.S.

(ii)

$$\text{L.H.S.} = \left(\frac{3}{5}\right)^{-4} \times \left(\frac{3}{5}\right)^{-3} = \frac{1}{\left(\frac{3}{5}\right)^4} \times \frac{1}{\left(\frac{3}{5}\right)^3} \qquad \left[\because a^{-n} = \frac{1}{a^n}\right]$$

$$= \frac{1}{\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}} = \frac{1}{\left(\frac{3}{5}\right)^7}$$

$$= \left(\frac{3}{5}\right)^{-7} = \left(\frac{5}{3}\right)^7 \qquad \left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$$

$$\text{R.H.S.} = \left(\frac{3}{5}\right)^{-4+(-3)} = \left(\frac{3}{5}\right)^{-7} = \left(\frac{5}{3}\right)^7 \qquad \left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$$

Hence, L.H.S. = R.H.S.

(iii)

$$\text{L.H.S.} = (-2)^5 \times (-2)^{-8}$$

$$= (-2)^5 \times \frac{1}{(-2)^8} \qquad \left[\because a^{-n} = \frac{1}{a^n}\right]$$

$$= \frac{(-2)^5}{(-2)^8} = \frac{(-2)\times(-2)\times(-2)\times(-2)\times(-2)}{(-2)\times(-2)\times(-2)\times(-2)\times(-2)\times(-2)\times(-2)\times(-2)} = \frac{1}{(-2)^3} = -\frac{1}{8}$$

$$\text{R.H.S.} = (-2)^{5+(-8)} = (-2)^{-3} = \frac{1}{(-2)^3} = -\frac{1}{8}$$

Hence, L.H.S. = R.H.S.

Example 8 : Evaluate :

(i) $6^7 \times 6^{-4}$ (ii) $3^{-9} \times 3^6$ (iii) $\left(\frac{5}{7}\right)^4 \times \left(\frac{5}{7}\right)^{-6}$

Solution : (i)

$$6^7 \times 6^{-4} = 6^{7+(-4)} = 6^{7-4} = 6^3 = 6 \times 6 \times 6 = 216 \qquad [\because a^m \times a^n = a^{m+n}]$$

(ii)

$$3^{-9} \times 3^6 = 3^{-9+6} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$$

(iii)

$$\left(\frac{5}{7}\right)^4 \times \left(\frac{5}{7}\right)^{-6} = \left(\frac{5}{7}\right)^{4+(-6)} = \left(\frac{5}{7}\right)^{4-6} = \left(\frac{5}{7}\right)^{-2} = \left(\frac{7}{5}\right)^2 = \frac{49}{25} \qquad \left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$$

Law II : If x is a non-zero rational number and m, n are any two integers, then $x^m \div x^n = x^{m-n}$ i.e., $\frac{x^m}{x^n} = x^{m-n}$.

Example 9 : Verify the following statements :

(i) $11^6 \div 11^4 = 11^{6-4}$ (ii) $5^4 \div 5^{-2} = 5^{4-(-2)}$ (iii) $7^{-4} \div 7^{-2} = 7^{-4-(-2)}$

Solution : (i)

$$\text{L.H.S.} = 11^6 \div 11^4 = \frac{11^6}{11^4}$$

$$= 11^6 \times 11^{-4} = 11^{6+(-4)} = 11^2 \qquad [\because a^m \times a^n = a^{m+n}]$$

$$\text{R.H.S.} = 11^{6-4} = 11^2$$

$\therefore$ L.H.S. = R.H.S.

(ii)

$$\text{L.H.S.} = 5^4 \div 5^{-2} = 5^4 \times \frac{1}{5^{-2}} = 5^4 \times 5^2 \qquad \left[\because \frac{1}{a^{-n}} = a^n\right]$$

$$= 5^{4+2} = 5^6 \qquad [\because a^m \times a^n = a^{m+n}]$$

$$\text{R.H.S.} = 5^{4-(-2)} = 5^{4+2} = 5^6$$

$\therefore$ L.H.S. = R.H.S.

(iii)

$$\text{L.H.S.} = 7^{-4} \div 7^{-2} = \frac{7^{-4}}{7^{-2}} = 7^{-4} \times 7^2 = 7^{-4+2} = 7^{-2} = \frac{1}{7^2} = \frac{1}{49} \qquad [\because a^m \times a^n = a^{m+n}]$$

$$\text{R.H.S.} = 7^{-4-(-2)} = 7^{-4+2} = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$$

$\therefore$ L.H.S. = R.H.S.

Example 10 : Evaluate using laws of exponents :

(i) $\left(\frac{21}{2}\right)^{-6} \div \left(\frac{21}{2}\right)^{-4}$ (ii) $(-2)^6 \div (-2)^9$ (iii) $\left(\frac{4}{3}\right)^3 \div \left(\frac{3}{4}\right)^{-6}$

Solution : (i)

$$\left(\frac{21}{2}\right)^{-6} \div \left(\frac{21}{2}\right)^{-4} = \left(\frac{21}{2}\right)^{-6-(-4)} \qquad [\because a^m \div a^n = a^{m-n}]$$

$$= \left(\frac{21}{2}\right)^{-6+4} = \left(\frac{21}{2}\right)^{-2} = \left(\frac{2}{21}\right)^2 = \frac{4}{441} \qquad \left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$$

(ii)

$$(-2)^6 \div (-2)^9 = (-2)^{6-9} \qquad [\because a^m \div a^n = a^{m-n}]$$

$$= (-2)^{-3} = \frac{1}{(-2)^3} = -\frac{1}{8}$$

(iii)

$$\left(\frac{4}{3}\right)^3 \div \left(\frac{3}{4}\right)^{-6} = \left(\frac{4}{3}\right)^3 \div \left(\frac{4}{3}\right)^6 \qquad \left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$$

$$= \left(\frac{4}{3}\right)^{3-6} = \left(\frac{4}{3}\right)^{-3} = \left(\frac{3}{4}\right)^3 = \frac{27}{64} \qquad \left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n\right]$$

Example 11 : Find x, if $6^{2x} \div 6^{-4} = 36$.

Solution :

$6^{2x} \div 6^{-4} = 36$

$\Rightarrow \quad 6^{2x} \div 6^{-4} = 6^2$

$\Rightarrow \quad 6^{2x-(-4)} = 6^2$

$\Rightarrow \quad 6^{2x+4} = 6^2$ $\quad [\because a^m = a^n, a > 0 \Rightarrow m = n]$

$\Rightarrow \quad 2x + 4 = 2$

$\Rightarrow \quad 2x = -2$

$\Rightarrow \quad x = -1$

Example 12 : Find the multiplicative inverse of $\left(\frac{3}{7}\right)^{13} \div \left(\frac{3}{7}\right)^{5}$.

Solution : $\left(\frac{3}{7}\right)^{13} \div \left(\frac{3}{7}\right)^{5} = \left(\frac{3}{7}\right)^{13-5} = \left(\frac{3}{7}\right)^{8}$ $\quad [\because a^m \div a^n = a^{m-n}]$

The multiplicative inverse of $\left(\frac{3}{7}\right)^{8}$ is $\left(\frac{3}{7}\right)^{-8} = \left(\frac{7}{3}\right)^{8}$. $\quad \left[\because \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}\right]$

EXERCISE 2.2

Use Cordova Smart Class Software on the smart board in class to do Exercise.

1. Simplify :
   (i) $3^{-4} \div 3^{5}$ (ii) $29^3 \div 29^2$ (iii) $\left(\frac{11}{9}\right)^{-7} \times \left(\frac{9}{11}\right)^{-8}$
2. Evaluate :
   (i) $(2^{-1} \times 4^{-1}) \div 2^{-2}$ (ii) $(3^{-3} \div 3^{-7}) \div 3^{-2}$ (iii) $(5^{-1} + 7^{0}) \times 5^3$
   (iv) $(2^{-1} \times 3^{-1})^{-1} \div 2$ (v) $\left(-\frac{1}{3}\right)^{-4} \div \left(-\frac{1}{3}\right)^{-2}$
3. Find the value of m, if :
   (i) $\left(\frac{3}{7}\right)^{-4} \times \left(\frac{3}{7}\right)^{10} = \left(\frac{3}{7}\right)^{2m}$ (ii) $\left(\frac{2}{9}\right)^{4} \times \left(\frac{2}{9}\right)^{-7} = \left(\frac{2}{9}\right)^{2m+1}$ (iii) $\left(\frac{3}{8}\right)^{2m} \times \left(\frac{3}{8}\right)^{6} \times \frac{3}{8} = \left(\frac{3}{8}\right)^{13}$
4. Simplify :
   (i) $(5^{-10} \div 5^{-7}) \div 5^{-4}$ (ii) $(3^8 \div 3^5) \times 3^{-3}$ (iii) $(-7)^4 \times \left(\frac{3}{7}\right)^{4} \div \frac{1}{3^{-5}}$
5. Evaluate :
   (i) $\left[\left(\frac{1}{2}\right)^{-3} - \left(\frac{1}{3}\right)^{-3}\right] \div \left(\frac{1}{4}\right)^{-3}$ (ii) $(2^{-1} \times 3^{-1})^{-1} \times 4^{-1}$ (iii) $(3^{-1} \times 5^{-1}) + 6^{-1}$
6. Find the multiplicative inverse of $\left(\frac{3}{5}\right)^{-3} \div \left(\frac{5}{3}\right)^{3}$.
7. Solve for x :
   (i) $\frac{7^3}{7^{x-1}} = 7^7$ (ii) $9^{2x} \div 9^{-5} = 9^{13}$
8. Simplify :
   (i) $\frac{15x^5y^{-2}z^4}{5x^3y^{-3}z^2}$ (ii) $\frac{42x^3y^2z^4}{6xy^2z^3}$ (iii) $\frac{3x^{-2}y^{-3}z}{9x^{-3}y^{-4}}$ (iv) $\frac{42x^3y^2z^4}{16x^2y^3z^8}$

9. Find the value of p, if $(-2)^{p+1} \cdot (-2)^{p} = -8$.
10. If $\frac{m}{n} = \left(\frac{3}{4}\right)^{5} \div \left(\frac{3}{4}\right)^{3}$, find $\left(\frac{n}{m}\right)^{-2}$.
11. Simplify :
    (i) $(-4)^{-2} \times \left(-\frac{3}{2}\right)^{-2}$ (ii) $\frac{2^{-3} \times 5^{-3} \times 10^2 \times 25}{5^4 \times 2^{-5}}$

Law III : If x is a non-zero rational number and m and n are any two integers, then $(x^m)^n = x^{mn}$.

Example 13 : Verify the following statements :

(i) $\left[\left(\frac{3}{5}\right)^{4}\right]^{-2} = \left(\frac{3}{5}\right)^{4\times(-2)}$ (ii) $(7^{-3})^{-2} = 7^{(-3)\times(-2)}$

Solution : (i) L.H.S. $= \left[\left(\frac{3}{5}\right)^{4}\right]^{-2} = \left[\left(\frac{3}{5}\right)^{4}\right]^{-1} \times \left[\left(\frac{3}{5}\right)^{4}\right]^{-1}$

$= \frac{1}{\left(\frac{3}{5}\right)^{4}} \times \frac{1}{\left(\frac{3}{5}\right)^{4}}$ $\quad \left[\because a^{-n} = \frac{1}{a^n}\right]$

$= \frac{1}{\left(\frac{3}{5}\right)^{4+4}}$ $\quad [\because a^m \times a^n = a^{m+n}]$

$= \frac{1}{\left(\frac{3}{5}\right)^{8}} = \left(\frac{3}{5}\right)^{-8}$ $\quad \left[\because \frac{1}{a^n} = a^{-n}\right]$

R.H.S. $= \left(\frac{3}{5}\right)^{4\times(-2)} = \left(\frac{3}{5}\right)^{-8}$

$\therefore$ L.H.S. = R.H.S.

(ii) L.H.S. $= (7^{-3})^{-2} = \frac{1}{(7^{-3})^2} = \frac{1}{7^{-3} \times 7^{-3}} = \frac{1}{7^{-3+(-3)}} = \frac{1}{7^{-6}} = 7^6$ $\quad \left[\because \frac{1}{a^{-n}} = a^n\right]$

R.H.S. $= 7^{(-3)\times(-2)} = 7^6$

$\therefore$ L.H.S. = R.H.S.

Law IV : *If x and y are non-zero rational numbers, m is any integer, then* $(x \times y)^m = x^m \times y^m$.

Example 14 : Verify the following statements :

(i) $(5 \times 6)^{-3} = 5^{-3} \times 6^{-3}$ (ii) $\left(\frac{2}{3} \times \frac{4}{5}\right)^{-3} = \left(\frac{2}{3}\right)^{-3} \times \left(\frac{4}{5}\right)^{-3}$

Solution : (i) L.H.S. $= (5 \times 6)^{-3} = \frac{1}{(5 \times 6)^3} = \frac{1}{5^3 \times 6^3} = \frac{1}{5^3} \times \frac{1}{6^3} = 5^{-3} \times 6^{-3}$ = R.H.S.

Hence, $(5 \times 6)^{-3} = 5^{-3} \times 6^{-3}$

(ii) L.H.S. $= \left(\frac{2}{3} \times \frac{4}{5}\right)^{-3} = \frac{1}{\left(\frac{2}{3} \times \frac{4}{5}\right)^{3}}$

$$= \frac{1}{\left(\frac{2}{3}\right)^3 \times \left(\frac{4}{5}\right)^3} = \frac{1}{\left(\frac{2}{3}\right)^3} \times \frac{1}{\left(\frac{4}{5}\right)^3}$$ [$\because (x \times y)^m = x^m \times y^m$]

$$= \left(\frac{2}{3}\right)^{-3} \times \left(\frac{4}{5}\right)^{-3}$$ [$\because \frac{1}{a^n} = a^{-n}$]

= R.H.S.

Hence, $\left(\frac{2}{3} \times \frac{4}{5}\right)^{-3} = \left(\frac{2}{3}\right)^{-3} \times \left(\frac{4}{5}\right)^{-3}$

Example 15 : Simplify :

(i) $(5a^2)^3$ (ii) $\frac{8x^3}{24(xy)^5}$ (iii) $\left(\frac{1}{6}\right)^{-2} + \left(\frac{1}{7}\right)^{-1} + 2^{-1}$

Solution : (i) $(5a^2)^3 = (5)^3 (a^2)^3$ [$\because (x \times y)^m = x^m \times y^m$]

$= 125\, a^{2 \cdot 3}$ [$\because (a^m)^n = a^{mn}$]

$= 125\, a^6$

(ii) $\frac{8x^3}{24(xy)^5} = \frac{8}{24} \times \frac{x^3}{x^5 \cdot y^5}$ [$\because (x \times y)^m = x^m \times y^m$]

$= \frac{1}{3} \times \frac{1}{x^5 \cdot x^{-3} \cdot y^5} = \frac{1}{3} \times \frac{1}{x^2 \cdot y^5} = \frac{1}{3x^2y^5}$ [$\because a^m \times a^n = a^{m+n}$]

(iii) $\left(\frac{1}{6}\right)^{-2} + \left(\frac{1}{7}\right)^{-1} + 2^{-1} = 6^2 + \left(\frac{7}{1}\right)^1 + \frac{1}{2}$ [$\because \frac{1}{a^{-n}} = a^n$]

$= 36 + 7 + \frac{1}{2} = 43 + \frac{1}{2} = \frac{87}{2}$

Example 16 : Solve for x: $3^{3x+3} = 9^{x+4}$.

Solution : $3^{3x+3} = 9^{x+4}$

$\Rightarrow 3^{3x+3} = (3^2)^{x+4}$

$\Rightarrow 3^{3x+3} = 3^{2x+8}$

$\Rightarrow 3x + 3 = 2x + 8$ [$\because a^m = a^n, a > 0 \Rightarrow m = n$]

$\Rightarrow 3x - 2x = 8 - 3$

$\Rightarrow x = 5$

Example 17 : Express the following with positive exponents :

(i) $\left(\frac{5}{6}\right)^{-7}$ (ii) $3^4 \times 3^{-9}$ (iii) $\left[\left(\frac{3}{4}\right)^{-3}\right]^{-2}$

Solution : (i) $\left(\frac{5}{6}\right)^{-7} = \frac{5^{-7}}{6^{-7}} = \frac{6^7}{5^7} = \left(\frac{6}{5}\right)^7$ (ii) $3^4 \times 3^{-9} = 3^{4+(-9)} = 3^{-5} = \frac{1}{3^5}$

(iii) $\left[\left(\frac{3}{4}\right)^{-3}\right]^{-2} = \left(\frac{3}{4}\right)^{(-3)(-2)} = \left(\frac{3}{4}\right)^6$

Example 18 : Simplify :

(i) $[(7)^{-3} \div (7)^{-8}] \times 7^{-6}$ (ii) $\frac{(8)^{-1} \times (5)^3}{5 \times (2)^{-4}}$

Solution : (i) $[(7)^{-3} \div (7)^{-8}] \times 7^{-6} = (7)^{-3-(-8)} \times (7)^{-6}$ [$\because a^m \div a^n = a^{m-n}$]

$= (7)^{-3+8} \times (7)^{-6}$ [$\because a^m \times a^n = a^{m+n}$]

$= (7)^5 \times (7)^{-6} = 7^{5+(-6)}$

$= 7^{-1} = \frac{1}{7}$

(ii) $\frac{(8)^{-1} \times (5)^3}{5 \times (2)^{-4}} = \frac{(2^3)^{-1} \times (5)^3}{5 \times (2)^{-4}}$

$= 2^{-3} \times 5^3 \times 5^{-1} \times 2^4 = 2^{-3+4} \times 5^{3+(-1)} = 2 \times 5^2 = 2 \times 25 = 50$

EXERCISE 2.3

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Simplify and write the answer in the form of $\frac{p}{q}$:

   (i) $\left[\left(\frac{1}{2}\right)^3\right]^2$ (ii) $(3^{-2})^3$

2. Find the value of $(6^{-1} - 7^{-1})^{-1}$.
3. Simplify :

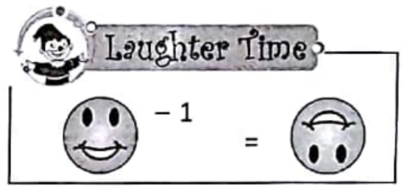
   (i) $6^0 - 7^0 - 5^0$ (ii) $(5^3)^2 \times 5^{-5}$ (iii) $(9^3)^2 \div (3^6)^2$

4. Simplify and write the answer with positive integral index :

   (i) $(-4 \times 7)^{-3}$ (ii) $27^3 \times 3^{-6}$ (iii) $\left(\frac{x^4\, y^3\, z^2}{xy^2\, z^4}\right)^{-2}$

5. By what number should $\left[\left(-\frac{5}{3}\right)^3\right]^{-3}$ be multiplied to obtain $\left(-\frac{3}{5}\right)^4$?
6. Simplify :

   (i) $(4^{-1} + 8^{-1}) \times \left(\frac{3}{2}\right)^{-1}$ (ii) $\left[\left(\frac{2}{3}\right)^2\right]^{-2}$



7. Find x, if $(3^{x+2} - 9) \div 8 = 9$.
8. Show that $\frac{25 \times 2x^{-4}}{5^{-2} \times 10x^{-6}} = 125x^2$.
9. Simplify :

   (i) $\left[\left(\frac{6}{7}\right)^{-1} - \left(\frac{1}{6}\right)^{-1}\right]^{-1} \div (29)^{-1}$ (ii) $\left[\left(\frac{1}{3}\right)^{-2} - \left(\frac{1}{2}\right)^{-3} + \left(\frac{1}{4}\right)^{-1}\right]^{-1} \div 5^{-2}$.

10. Find the value of m, if $(-5)^{m+1} \times (-5)^6 = (-1)^9\, (5)^9$.

USE OF EXPONENTS TO EXPRESS THE NUMBERS IN THE STANDARD FORM

The speed of light is 300,000,000 m/sec.

We can write this number as 3×10^8 m/sec. Similarly, $0.0000009 = \frac{9}{10000000} = \frac{9}{10^7} = 9 \times 10^{-7}$.

Numbers in Standard Form

A number written as $a \times 10^b$ (read as a times ten raised to the power of b) is said to be in standard form, if b is an integer and a is a decimal number such that $1 \le a < 10$.

Rule : *If the number is less than 1, then the decimal point is moved to the right so that there remains only one digit on the left of the decimal.*

The given number is written as the product of the number so obtained and $(10)^{-n}$, where n is the number of places, the decimal point has been moved to the right.

Example 19 : Write the following numbers in standard form :

(*i*) 0.0000008 (*ii*) 0.00016 (*iii*) 16250000000

Solution : (*i*) $0.0000008 = \frac{8}{10000000} = \frac{8}{10^7} = 8 \times 10^{-7}$

(*ii*) $0.00016 = \frac{16}{100000} = \frac{1.6 \times 10}{100000} = \frac{1.6}{10000} = 1.6 \times 10^{-4}$

(*iii*) $16250000000 = 1625 \times 10000000 = 1.625 \times 1000 \times 10000000 = 1.625 \times 10^3 \times 10^7 = 1.625 \times 10^{10}$

Example 20 : Express the following in usual form :

(*i*) 4.23×10^{-5} (*ii*) 6×10^{-8} (*iii*) 1.00001×10^9 (*iv*) 7.6×10^{12}

Solution : (*i*) $4.23 \times 10^{-5} = \frac{423}{100} \times 10^{-5} = \frac{423}{10^2 \times 10^5} = \frac{423}{10^7} = \frac{423}{10000000} = 0.0000423$

(*ii*) $6 \times 10^{-8} = \frac{6}{10^8} = \frac{6}{100000000} = 0.00000006$

(*iii*) $1.00001 \times 10^9 = \frac{100001}{100000} \times 10^9 = \frac{100001 \times 10^9}{10^5} = 100001 \times 10^4 = 1000010000$

(*iv*) $7.6 \times 10^{12} = \frac{76}{10} \times 10^{12} = 76 \times 10^{11} = 7600000000000$

Example 21 : Express the number appearing in the following statements in standard form :

(*i*) Charge of an electron is 0.00000000000000000016 coulomb.

(*ii*) 1 micron is equal to $\frac{1}{1000000}$ m.

(*iii*) Size of a bacteria is 0.0000005 m.

(*iv*) Thickness of a thick paper is 0.07 mm.

Solution : (*i*) Charge of an electron = 0.00000000000000000016 coulomb

$= \frac{16}{10^{20}}$ coulomb $= \frac{1.6 \times 10}{10^{20}}$ coulomb $= \frac{1.6}{10^{19}}$ coulomb $= 1.6 \times 10^{-19}$ coulomb

(*ii*) 1 micron is equal to $\frac{1}{1000000}$ m $= \frac{1}{10^6}$ m $= 10^{-6}$m

(*iii*) Size of a bacteria is 0.0000005 m $= \frac{5}{10000000}$ m $= \frac{5}{10^7}$ m $= 5 \times 10^{-7}$ m

(*iv*) Thickness of a thick paper is 0.07 mm $= \frac{7}{100}$ mm $= \frac{7}{10^2}$ mm $= 7 \times 10^{-2}$ mm

EXERCISE 2.4

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Write the following numbers in standard form :
   (*i*) 0.0000008 (*ii*) 37400000000 (*iii*) 0.00005
   (*iv*) 0.00000007 (*v*) 25430000 (*vi*) 0.00018
2. Write the following numbers in usual form :
   (*i*) 6.5×10^5 (*ii*) 4×10^{-6} (*iii*) 4.123×10^6
   (*iv*) 3.2×10^{-4} (*v*) 7.54×10^{-5}
3. The diameter of the Sun is 1.4×10^9m whereas the diameter of the Earth is 1.2756×10^7m. Compare the diameter of the Sun, with the diameter of the Earth.
4. The size of a red blood cell is 0.000007 m and that of a plant cell is 0.00001275 m. What is the approximate ratio of their sizes?

FACTS TO REMEMBER

- Laws of exponents :
  (*i*) $a^m \times a^n = a^{m+n}$ (*ii*) $a^m \div a^n = a^{m-n}$ (*iii*) $(a^m)^n = a^{mn}$
  (*iv*) $a^m \times b^m = (ab)^m$ (*v*) $a^0 = 1$ (*vi*) $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$ (*vii*) $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
- Very small numbers can be expressed in standard form using negative exponents.

HOTS QUESTIONS

1. If $4^{x+1} - 4^x = 24$, what is the value of $(2x)^x$?
2. Solve for x : $\frac{5^{2x} \times 5}{5^{3x-2}} = (25)^{-2}$

ANSWERS

Exercise 2.1

1. (*i*) -32 (*ii*) $\frac{-125}{216}$ (*iii*) $\frac{1}{16}$ (*iv*) $\frac{1}{256}$ (*v*) $\frac{25}{16}$
2. (*i*) 49 (*ii*) $-\frac{1}{8}$ (*iii*) $\frac{1}{25}$ (*iv*) $\frac{9}{16}$
4. $\frac{-1}{4}$ 5. $\frac{-2}{9}$
6. (*i*) 1 (*ii*) $\frac{4}{7}$ (*iii*) $\frac{1}{3}$ (*iv*) $\frac{49}{36}$
7. (*i*) $\frac{-25}{8}$ (*ii*) 4 (*iii*) 3

Exercise 2.2

1. (*i*) 3 (*ii*) 29 (*iii*) $\frac{11}{9}$
2. (*i*) $\frac{1}{2}$ (*ii*) 81 (*iii*) 150 (*iv*) 3 (*v*) 9

3. (i) 2 (ii) – 2 (iii) 3

4. (i) 5 (ii) 1 (iii) $\frac{1}{3}$

5. (i) $-\frac{19}{64}$ (ii) $\frac{3}{2}$ (iii) $\frac{2}{5}$

6. $\frac{25}{9}$ 7. (i) –2 (ii) 4

8. (i) $3x^2yz^2$ (ii) $7x^2z$ (iii) $\frac{1}{3}yz$ (iv) $\frac{21x}{8yz^4}$

9. 1 10. $\frac{81}{256}$ 11. (i) $\frac{1}{36}$ (ii) $\frac{16}{125}$

Exercise 2.3

1. (i) $\frac{1}{64}$ (ii) $\frac{1}{729}$ 2. 42

3. (i) –1 (ii) 5 (iii) 1

4. (i) $\frac{1}{(-28)^3}$ (ii) 27 (iii) $\frac{z^4}{x^6y^2}$ 5. $\left(-\frac{5}{3}\right)^5$

6. (i) $\frac{1}{4}$ (ii) $\frac{81}{16}$ 7. 2

9. (i) –6 (ii) 5 10. 2

Exercise 2.4

1. (i) 8×10^{-7} (ii) 3.74×10^{10} (iii) 5×10^{-5} (iv) 7×10^{-4}
   (v) 2.543×10^{7} (vi) 1.8×10^{-4}
2. (i) 650000 (ii) 0.000004 (iii) 4123000
   (iv) 0.00032 (v) 0.0000754
3. 100 times (approx.)
4. 1 : 2

HOTS

1. $3^{1/2}$
2. 7

3 Squares and Square Roots

Use **Cordova Smart Class Software** on the smart board in class to learn about squares and square roots.

SQUARE

The square of a number is the product of the number with itself.

For example : The square of 2 is $2^2 = 2 \times 2 = 4$.
The square of 3 is $3^2 = 3 \times 3 = 9$.
The square of 4 is $4^2 = 4 \times 4 = 16$.
The square of 5 is $5^2 = 5 \times 5 = 25$.

Perfect Squares

When the exponent (power) of a natural number is 2, the number so obtained is called a **square number** or a **perfect square**. Thus, when a number is multiplied by itself, the product is a perfect square.

For example : $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, $25 = 5^2$. Therefore, 1, 4, 9, 16, 25 are perfect squares.

To Find Whether a Number is a Perfect Square or Not

(a) Write the prime factorisation of the given number.
(b) Group the prime factors in such a way that in each pair, both factors are same.
(c) If no factor is left over after grouping, the number is a perfect square.
(d) If any factor (or factors) are left whose grouping is not possible, then the number is not a perfect square.

Example 1 : Check whether the following numbers are perfect squares :
(i) 14641 (ii) 7688

Solution : (i)

$14641 = 11 \times 11 \times 11 \times 11$
$= (11 \times 11) \times (11 \times 11)$
$= (11)^2 \times (11)^2$
$= (11 \times 11)^2 = (121)^2$

11	14641
11	1331
11	121
11	11
	1

Since, no factor is left over, therefore 14641 is a perfect square.

(ii)

$7688 = 2 \times 2 \times 2 \times 31 \times 31$
$= (2 \times 2) \times (31 \times 31) \times 2$
$= (2)^2 \times (31)^2 \times 2$

2	7688
2	3844
2	1922
31	961
31	31
	1

Since 2 cannot be grouped, therefore 7688 is not a perfect square of any natural number.

Example 2 : Find the smallest number by which 1800 must be multiplied so that the product is a perfect square.

Solution :

$1800 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$
$= (2 \times 2) \times (3 \times 3) \times (5 \times 5) \times 2$
$= 2^2 \times 3^2 \times 5^2 \times 2$

2	1800
2	900
2	450
3	225
3	75
5	25
5	5
	1

For a number to be a perfect square, it should make pairs of all its prime factors. Here, 2 is left without a pair, hence, we must multiply 1800 by 2 to make it a perfect square.

Hence, the smallest number by which 1800 must be multiplied to make it a perfect square is 2.

Example 3 : Find the smallest number by which 4056 be divided to make it a perfect square.

Solution :

$4056 = 2 \times 2 \times 2 \times 3 \times 13 \times 13$
$= (2 \times 2) \times (13 \times 13) \times 2 \times 3$

2	4056
2	2028
2	1014
3	507
13	169
13	13
	1

In this case, two factors 2 and 3 are left for which grouping is not possible. Hence, to get a perfect square (2×3) must be eliminated. So, we should divide 4056 by $2 \times 3 = 6$ to get a number which is a perfect square.

PROPERTIES OF SQUARE NUMBERS

Consider the table below which shows squares of all natural numbers from 1 to 10 :

Number (x)	Square of a number (x^2)		Perfect Square
1	1		1
2	$4 = 3 \times 1 + 1$	$= 4 \times 1$	4
3	$9 = 3 \times 3$	$= 4 \times 2 + 1$	9
4	$16 = 3 \times 5 + 1$	$= 4 \times 4$	16
5	$25 = 3 \times 8 + 1$	$= 4 \times 6 + 1$	25
6	$36 = 3 \times 12$	$= 4 \times 9$	36
7	$49 = 3 \times 16 + 1$	$= 4 \times 12 + 1$	49
8	$64 = 3 \times 21 + 1$	$= 4 \times 16$	64
9	$81 = 3 \times 27$	$= 4 \times 20 + 1$	81
10	$100 = 3 \times 33 + 1$	$= 4 \times 25$	100

By observing the above table closely, we conclude the following properties:

Property I : (*i*) *No perfect square end with 2, 3, 7 or 8 at the units place.*

(*ii*) *A perfect square ends with 0, 1, 4, 5, 6 or 9 at the units place.*

Example 4 : Show that the following natural numbers are not perfect squares :

(*i*) 1057 (*ii*) 23453 (*iii*) 7928 (*iv*) 2222
(*v*) 88733 (*vi*) 1537

Solution : We know that the natural numbers ending with the digits 2 , 3, 7 or 8 are not perfect squares.

(*i*) 1057 ends with digit 7. (*ii*) 23453 ends with digit 3. (*iii*) 7928 ends with digit 8.
(*iv*) 2222 ends with digit 2. (*v*) 88733 ends with digit 3. (*vi*) 1537 ends with digit 7.

Therefore, all these numbers are not perfect squares.

Property II : (*i*) *If a number has 1 or 9 at the units place, then its square ends with 1.*

(*ii*) *If a number has 4 or 6 at the units place, then its square ends with 6.*

Example 5 : What will be the units digits in the square of the following numbers?

(*i*) 31 (*ii*) 722 (*iii*) 2873
(*iv*) 2968 (*v*) 21976

Solution : (*i*) Units digit of $(31)^2$ is $1^2 = 1$.
(*ii*) Units digit of $(722)^2$ is $2^2 = 4$.
(*iii*) Units digit of $(2873)^2$ is $3^2 = 9$.
(*iv*) Units digit of $(2968)^2$ is $8^2 = 4$.
($\because$ Units digit of $8^2 = 64$ is 4)
(*v*) Units digit of $(21976)^2$ is $6^2 = 6$.
($\because$ Units digit of $6^2 = 36$ is 6)

Property III : *A number ending in an odd number of zeros is never a perfect square.*

Example 6 : Are the following numbers perfect squares?

(*i*) 64000 (*ii*) 505050 (*iii*) 33000

Solution : (*i*) 64000 has 3 (odd number) zeros at the end.
(*ii*) 505050 has 1 (odd number) zero at the end.
(*iii*) 33000 has 3 (odd number) zeros at the end.

Hence, these are not perfect squares.

Property IV : *Between the squares of the numbers n and n + 1, there are 2n non-perfect square numbers.*

For example : Square numbers are $1^2, 2^2, 3^2, 4^2$ ...

(*i*) Between 1^2 and 2^2, there are $2 \times 1 = 2$ non-perfect square numbers, *viz.*, 2, 3.
(*ii*) Between 4^2 and 5^2, there are $4 \times 2 = 8$ non-perfect square numbers, *viz.*, 17, 18, 19, 20, 21, 22, 23, 24.
(*iii*) Between 6^2 and 7^2, there are $2 \times 6 = 12$ non-perfect square numbers, *viz.*, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48.

Example 7 : How many non-perfect square numbers lie between the squares of the following numbers?

(*i*) 12 and 13 (*ii*) 5 and 6

Solution : (*i*) Between $(12)^2$ and $(13)^2$, there are $2 \times 12 = 24$ non-perfect square numbers.
(*ii*) Between 5^2 and 6^2, there are $2 \times 5 = 10$ non-perfect square numbers.

Property V : *The square of an even number is always an even number and the square of an odd number is always an odd number.*

Example 8 : Determine whether the square of each of the following numbers is even or odd.

(*i*) 2826 (*ii*) 283 (*iii*) 79 (*iv*) 122

Solution : (*i*) 2826 being an even number, $(2826)^2$ will also be an even number.
(*ii*) 283 being an odd number, $(283)^2$ will also be an odd number.
(*iii*) 79 being an odd number, $(79)^2$ will also be an odd number.
(*iv*) 122 being an even number, $(122)^2$ will also be an even number.

Property VI : (*i*) *The square of a natural number (except 1) is either a multiple of 3 or exceeds a multiple of 3 by 1.*

(*ii*) *The square of a natural number (except 1) is either a multiple of 4 or exceeds a multiple of 4 by 1. (Look at the table of squares of natural numbers)*

Property VII : *The square of a natural number ending with 5 follows a definite pattern.*

$(25)^2 = (2 \times 3)$ hundred $+ 25 = 625$
$(35)^2 = (3 \times 4)$ hundred $+ 25 = 1225$
$(45)^2 = (4 \times 5)$ hundred $+ 25 = 2025$

Example 9 : Without actual multiplication, find the square of :

(i) 75 (ii) 95 (iii) 105

Solution : (i) $(75)^2 = (7 \times 8)$ hundred + 25 = 5625

(ii) $(95)^2 = (9 \times 10)$ hundred + 25 = 9025

(iii) $(105)^2 = (10 \times 11)$ hundred + 25 = 11025

Property VIII : *The sum of first n odd natural numbers is n^2.*

For example : First odd number $= 1 = 1^2$

Sum of first two odd numbers $= 1 + 3 = 4 = 2^2$

Sum of first three odd numbers $= 1 + 3 + 5 = 9 = 3^2$

Sum of first four odd numbers $= 1 + 3 + 5 + 7 = 16 = 4^2$

In other words, *if a number is a perfect square, then it is always equal to the sum of consecutive odd numbers starting from 1.*

Example 10 : Without adding, find the sum :

(i) 1 + 3 + 5 + 7 + 9 (ii) 1 + 3 + 5 + 7 + ... + 17 + 19 (iii) 1 + 3 + 5 + 7 + ... + 19 + 21

Solution : (i) 1 + 3 + 5 + 7 + 9 is the sum of first five odd natural numbers, therefore, the sum $5^2 = 25$.

(ii) 1 + 3 + 5 + 7 + ... + 17 + 19 is the sum of first 10 odd natural numbers, therefore, the sum will be $(10)^2 = 100$.

(iii) 1 + 3 + 5 + 7 + ... + 19 + 21 is the sum of first 11 odd natural numbers, therefore, the sum will be $(11)^2 = 121$.

Property IX : Some square numbers follow interesting patterns. Observe the following :

(i) $(1)^2 = 1$

$(11)^2 = 121$

$(111)^2 = 12321$

$(1111)^2 = 1234321$

$(11111)^2 = 123454321$

....

....

The sum of the digits of every number on the right hand side is a perfect square.

121	= 1 + 2 + 1	= 4	$= 2^2$
12321	= 1 + 2 + 3 + 2 + 1	= 9	$= 3^2$
1234321	= 1 + 2 + 3 + 4 + 3 + 2 + 1	= 16	$= 4^2$
123454321	= 1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1	= 25	$= 5^2$
....	....	....	....
....	....	....	....

(ii) $7^2 = 49$

$(67)^2 = 4489$

$(667)^2 = 444889$

$(6667)^2 = 44448889$

....

....

Using this pattern, we can find the squares of the numbers 66667, 666667 ... .

EXERCISE 3.1

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Show that the following numbers are not perfect squares :
   (i) 54473 (ii) 4058 (iii) 24257 (iv) 3332
2. What will be the units digits of the squares of the following numbers?
   (i) 1234 (ii) 4329 (iii) 8723
3. Which of the following end with digit 1?
   (i) $(37)^2$ (ii) $(321)^2$ (iii) $(549)^2$ (iv) $(427)^2$
4. Determine whether the squares of the following numbers are odd or even :
   (i) 2826 (ii) 7779 (iii) 30018 (iv) 8204
5. How many natural numbers lie between the squares of the following numbers?
   (i) 25 and 26 (ii) 19 and 20
6. Find the squares of the following numbers without actual multiplication.
   (i) 65 (ii) 95 (iii) 205

PYTHAGOREAN TRIPLET

If a, b, c are three numbers where $c > a$ and $c > b$ such that $a^2 + b^2 = c^2$, then (a, b, c) is called Pythagorean triplet.

For example : (3, 4, 5), (6, 8, 10) and (5, 12, 13) are Pythagorean triplets, because

(i) $(3)^2 + (4)^2 = (5)^2$ (ii) $(6)^2 + (8)^2 = (10)^2$ (iii) $(5)^2 + (12)^2 = (13)^2$

For any number $m > 1$, $2m$, m^2-1 and $m^2 + 1$ is a Pythagorean triplet.

Example 11 : Write a Pythagorean triplet whose one member is :

(i) 14 (ii) 16

Solution : We know that, $2m$, $m^2 - 1$, $m^2 + 1$ is a Pythagorean triplet.

(i) Let $2m = 14$

$\Rightarrow m = 7$

$\therefore m^2 - 1 = 7^2 - 1 = 48$

and $m^2 + 1 = 7^2 + 1 = 50$

Hence, 14, 48, 50 is a Pythagorean triplet.

(ii) Let $2m = 16$

$\Rightarrow m = 8$

$\therefore m^2 - 1 = 8^2 - 1 = 63$

and $m^2 + 1 = 8^2 + 1 = 65$

Hence, 16, 63, 65 is a Pythagorean triplet.

EXERCISE 3.2

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Write a Pythagorean triplet whose smallest member is 8.
2. Find a Pythagorean triplet whose one member is 12.
3. One member of a Pythagorean triplet is 18. Find other two members of the triplet.
4. Is 10, 24 and 26 a Pythagorean triplet?

SQUARE ROOTS

We know that,

4 is the square of 2 as $4 = 2^2$.

9 is the square of 3 as $9 = 3^2$.
16 is the square of 4 as $16 = 4^2$.
25 is the square of 5 as $25 = 5^2$.

Now, consider its reverse.

We say that,

2 is the square root of 4.
3 is the square root of 9.
4 is the square root of 16.
5 is the square root of 25.

The square root of a number x is that number which when multiplied by itself gives x as the product.

We denote the square root of a number x by $\sqrt{x}$.

For example : Square root of 25 is represented as $\sqrt{25} = 5$.

Square root of 36 is represented as $\sqrt{36} = 6$.

Square root of 49 is represented as $\sqrt{49} = 7$.

Square root of 64 is represented as $\sqrt{64} = 8$.

To find the square root of a given number, we have to find a number which when multiplied by itself gives the given number.

For example : The square root of 25 = $\sqrt{25}$ = 5 as $5 \times 5 = 5^2 = 25$.

Though $(-5) \times (-5)$ also gives 25, but we will consider only positive numbers.

Properties of Square Roots

(*i*) *The square root of an even square number is an even number and the square root of an odd square number is an odd number.*

Square root of even square number	Square root of odd square number
$\sqrt{4} = 2$	$\sqrt{9} = 3$
$\sqrt{16} = 4$	$\sqrt{25} = 5$
$\sqrt{36} = 6$	$\sqrt{49} = 7$
$\sqrt{64} = 8$	$\sqrt{81} = 9$

(*ii*) *If a number has a natural number as square root, then its units digit must be 0, 1, 4, 5, 6 or 9.*

Square root of negative numbers are imaginary numbers and we will study such cases in higher classes.

We shall now discuss the following methods to determine the square roots :

- Prime Factorisation Method
- Successive (repeated) Subtraction Method
- Division Method

To Find Square Root by Prime Factorisation Method

By definition, square root of 9 = $\sqrt{9} = \sqrt{3 \times 3} = 3$

square root of 16 = $\sqrt{16} = \sqrt{4 \times 4} = 4$

square root of 25 = $\sqrt{25} = \sqrt{5 \times 5} = 5$

Considering the above three square roots, we can find the square root of a perfect square by using the following steps :

Step 1 : Write the prime factorisation of the given number.

Step 2 : The number being the perfect square, will have one or more pairs of the same prime factors. Write one factor from each pair.

Step 3 : Multiply the factors selected. The product will be the square root of the number.

Example 12 : Find the square root of

(*i*) 625 (*ii*) 1225 (*iii*) 50625

Solution : (*i*) $625 = 5 \times 5 \times 5 \times 5$

$\therefore \sqrt{625} = \sqrt{\underline{5 \times 5} \times \underline{5 \times 5}}$

$= 5 \times 5 = 25$

5	625
5	125
5	25
5	5
	1

(*ii*) $1225 = 5 \times 5 \times 7 \times 7$

$\therefore \sqrt{1225} = \sqrt{\underline{5 \times 5} \times \underline{7 \times 7}}$

$= 5 \times 7$

$= 35$

5	1225
5	245
7	49
7	7
	1

(*iii*) $50625 = 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5 \times 5$

$\therefore \sqrt{50625} = \sqrt{\underline{3 \times 3} \times \underline{3 \times 3} \times \underline{5 \times 5} \times \underline{5 \times 5}}$

$= 3 \times 3 \times 5 \times 5$

$= 225$

3	50625
3	16875
3	5625
3	1875
5	625
5	125
5	25
5	5
	1

Example 13 : Find the square root of the following numbers by prime factorisation method :

(*i*) 38416 (*ii*) 47089

Solution : (*i*) $38416 = 2 \times 2 \times 2 \times 2 \times 7 \times 7 \times 7 \times 7$

$\therefore \sqrt{38416} = \sqrt{\underline{2 \times 2} \times \underline{2 \times 2} \times \underline{7 \times 7} \times \underline{7 \times 7}}$

$= 2 \times 2 \times 7 \times 7$

$= 196$

2	38416
2	19208
2	9604
2	4802
7	2401
7	343
7	49
7	7
	1

(*ii*) $47089 = 7 \times 7 \times 31 \times 31$

$\therefore \sqrt{47089} = \sqrt{\underline{7 \times 7} \times \underline{31 \times 31}}$

$= 7 \times 31$

$= 217$

7	47089
7	6727
31	961
31	31
	1

To Find Square Root by Successive Subtraction Method

We know that, sum of the first n odd natural numbers is n^2.

i.e., $1 + 3 + 5 + 7 + 9 + ... + (2n - 1) = n^2$

With the help of this we can find the square root of smaller natural numbers. This method is time consuming for finding square roots of large natural numbers.

Step 1 : From the given number, whose square root is to be determined, subtract successively odd numbers 1, 3, 5, 7, 9, ... and so on.

Step 2 : Keep on performing subtraction till we get zero.

Step 3 : The number of times we have to perform subtraction will be the square root of the given number.

Example 14 : Find the square root of 49 by repeated subtraction.

Solution : We have, 49 – 1 = 48, 48 – 3 = 45,
45 – 5 = 40, 40 – 7 = 33, 13 – 13 = 0.
33 – 9 = 24, 24 – 11 = 13,

We have performed subtraction seven times. Hence, $\sqrt{49} = 7$.

Example 15 : Find by successive subtraction, the square root of 121.

Solution : 121 – 1 = 120, 120 – 3 = 117, 117 – 5 = 112, 112 – 7 = 105,
105 – 9 = 96, 96 – 11 = 85, 85 – 13 = 72, 72 – 15 = 57,
57 – 17 = 40, 40 – 19 = 21, 21 – 21 = 0.

We have performed subtraction 11 times.

Hence, $\sqrt{121} = 11$.

Example 16 : What would be the possible units digit of the square root of each of the following numbers? Also state whether the square root will be an even or an odd number.

(*i*) 5329 (*ii*) 15876 (*iii*) 105625 (*iv*) 8649 (*v*) 16641

Solution : (*i*) Since, units digit of 5329 is 9, the possible units digit of the square root may be 3 or 7. Also, the square root will be an odd number.

(*ii*) Since, units digit of 15876 is 6, the square root may have 4 or 6 at its units place. Also, the square root will be an even number.

(*iii*) Since, units digit of 105625 is 5, its square root will have 5 at its units place. Also, the square root will be an odd number.

(*iv*) Since, the units digit of 8649 is 9, its square root will have 3 or 7 at its units place. Also, the square root will be an odd number.

(*v*) Since, 16641 has 1 at its units place, so, its square root will have 1 or 9 at the units place. Also, the square root will be an odd number.

EXERCISE 3.3

Use Cordova Smart Class Software on the smart board in class to do Exercise.

1. What would be the possible ones digit of the square root of each of the following natural numbers?
   (*i*) 6561 (*ii*) 24336 (*iii*) 16129 (*iv*) 160801
2. Find the square root of the following by repeated subtraction method.
   (*i*) 144 (*ii*) 289 (*iii*) 169
3. Find the square root by prime factorisation method.
   (*i*) 676 (*ii*) 1024 (*iii*) 27225 (*iv*) 7744
   (*v*) 9604 (*vi*) 15625 (*vii*) 390625 (*viii*) 9801

Word Problems on Squares and Square Roots

Example 17 : Find the smallest number by which 1620 should be divided to get a perfect square. Find the square root of the square number so obtained.

Solution : $1620 = \underline{2\times2}\times\underline{3\times3}\times\underline{3\times3}\times5$

2	1620
2	810
3	405
3	135
3	45
3	15
5	5
	1

The prime factor 5 is left unpaired. So, 1620 must be divided by 5 to get the square number.

$\therefore \quad \frac{1620}{5} = 324$

Now, $\sqrt{324} = \sqrt{2\times2\times3\times3\times3\times3} = 2\times3\times3 = 18$

Example 18 : The product of two numbers is 972 and their quotient is $\frac{4}{3}$. Find the numbers.

Solution : Let one number be a, then the other number is $\frac{972}{a}$.

Given that : $\frac{a}{\frac{972}{a}} = \frac{4}{3}$

$\Rightarrow \quad \frac{a^2}{972} = \frac{4}{3}$

$\Rightarrow \quad a^2 = 972 \times \frac{4}{3} = 1296$

$\Rightarrow \quad a = \sqrt{1296} = \sqrt{\underline{2\times2}\times\underline{2\times2}\times\underline{3\times3}\times\underline{3\times3}}$

$\Rightarrow \quad a = 2\times2\times3\times3$

$\Rightarrow \quad a = 36$

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

One number is 36.

The other number is $\frac{972}{36} = 27$.

Hence, the numbers are 36 and 27.

Example 19 : Find the smallest square number which is divisible by each of the numbers 8, 15 and 20.

Solution : The least number divisible by each of the numbers 8, 15 and 20 is their L.C.M.

L.C.M. of 8, 15 and 20 $= 2\times2\times5\times2\times3$
$= 120$

2	8, 15, 20
2	4, 15, 10
5	2, 15, 5
	2, 3, 1

Now, $120 = \underline{2\times2}\times5\times2\times3$

The prime factors 5, 2 and 3 are not in pairs. For the number to be a perfect square, each factor of the number must have a pair. To make pairs of 5, 2 and 3, the number 120 has to be multiplied by 5 × 2 × 3 = 30.

Hence, the required square number is 120 × 30 = 3600.

Example 20 : The product of two numbers is 10625. If one number is 17 times the other number, find the numbers.

Solution : Let one number be x.

Then, the other number is $17x$.

$\therefore \quad x \times 17x = 10625$

$\Rightarrow \quad 17x^2 = 10625$

$\Rightarrow \quad x^2 = \frac{10625}{17} = 625$

$\Rightarrow \quad x = \sqrt{625}$

$\Rightarrow \quad x = \sqrt{\underline{5\times5}\times\underline{5\times5}} = 5 \times 5 = 25$

$$\begin{array}{r|r} 5 & 625 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$\therefore$ One number is 25 and other number is $17 \times 25 = 425$.

So, the numbers are 25 and 425.

Example 21 : In a farm, 7056 trees have been planted in such a way that there are as many trees in a row as there are rows in the farm. In how many rows the trees have been planted?

Solution : Let there be n rows of trees.

$\therefore$ Number of trees in each row = n

$\therefore \quad n \times n = 7056$

$\Rightarrow \quad n^2 = 7056$

$\Rightarrow \quad n = \sqrt{7056}$

$= \sqrt{\underline{2\times2}\times\underline{2\times2}\times\underline{3\times3}\times\underline{7\times7}}$

$= 2 \times 2 \times 3 \times 7 = 84$

$$\begin{array}{r|r} 2 & 7056 \\ \hline 2 & 3528 \\ \hline 2 & 1764 \\ \hline 2 & 882 \\ \hline 3 & 441 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

The number of rows is 84.

EXERCISE 3.4

Use Cordova Smart Class Software on the **smart board** in class to do Exercise.

1. Find the smallest square number which is divisible by each of the numbers 6, 9 and 15.
2. Find the smallest number by which 5392 must be divided so that the quotient is a perfect square. Find the square root of the quotient.
3. The product of two numbers is 2197. If one of the numbers is 13 times the other number, find the numbers.
4. A rectangular field is 81 m long and 49 m wide. A square field has the same area as that of the rectangular field. Find the side of the square field.
5. A school collected ₹ 11025 as fees from its students. If each student paid as much money as there were students in the school, how many students were there in the school?
6. Find the smallest number by which 27783 must be multiplied so that it becomes a perfect square.
7. The product of two numbers is 4046 and their quotient is $\frac{7}{2}$. Find the numbers.
8. For each of the following numbers, find the smallest number by which it should be divided so as to get a perfect square. Also, find the square root of the square number so obtained.
   (*i*) 2925 (*ii*) 2800 (*iii*) 2645

To Find Square Root by Division Method

When the numbers are large, even the method of finding square roots by prime factorisation method becomes lengthy and difficult. By division method, square roots can be easily determined.

Let us find the square root of 676 by division method.

Step 1 : Place a bar over every pair of digits starting from the digit at units place. If the number of digits in the given number is odd, then the single digit, which is left, will also be under the bar.

$$\overline{6}\,\overline{76}$$

Step 2 : Find the largest number whose square is less than or equal to the number at the extreme left under the bar. Let this number be the divisor and the quotient. So, 2 (here) is the divisor as well as the quotient.

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{6}\,\overline{76} \\ & 4 \end{array}$$

Step 3 : Subtract the product of the divisor and the quotient from the number at the extreme left under the bar to get the remainder (2 in this case).

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{6}\,\overline{76} \\ & 4 \\ \hline & 2 \end{array}$$

Step 4 : Copy the other pair under the bar to the right of the remainder. This becomes the new dividend. In this case, it is 276.

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{6}\,\overline{76} \\ & 4 \\ \hline & 276 \end{array}$$

Step 5 : Double the divisor ($2 \times 2 = 4$) and write it with a blank (□) on its right.

$$\begin{array}{r|l} & 2 \\ \hline 2 & \overline{6}\,\overline{76} \\ & 4 \\ \hline 4\square & 276 \end{array}$$

Step 6 : Guess a largest possible digit to fill the blank (step 5) which will also become the new digit in the quotient in such a way that the product of new divisor and the new quotient is equal to or less than the new dividend.

$$\begin{array}{r|l} & 26 \\ \hline 2 & \overline{6}\,\overline{76} \\ & 4 \\ \hline 46 & 276 \\ & 276 \end{array}$$

Step 7 : Subtract to get the remainder. See that the remainder is zero and no digit (pair) of given number under the bar is left. Therefore,

$\sqrt{676} = 26$

$$\begin{array}{r|l} & 26 \\ \hline 2 & \overline{6}\,\overline{76} \\ & 4 \\ \hline 46 & 276 \\ & 276 \\ \hline & \times \end{array}$$

We continue this process till all the pairs are used.

Example 22 : Find the square root of the following by division method :

(*i*) 4489 (*ii*) 7921 (*iii*) 5776

Solution : (*i*)

$$\begin{array}{r|l} & 67 \\ \hline 6 & \overline{44}\,\overline{89} \\ & 36 \\ \hline 127 & 8\,89 \\ & 8\,89 \\ \hline & 0 \end{array}$$

$\sqrt{4489} = 67$

(*ii*)

$$\begin{array}{r|l} & 89 \\ \hline 8 & \overline{79}\,\overline{21} \\ & 64 \\ \hline 169 & 15\,21 \\ & 15\,21 \\ \hline & 0 \end{array}$$

$\sqrt{7921} = 89$

(*iii*)

$$\begin{array}{r|l} & 76 \\ \hline 7 & \overline{57}\,\overline{76} \\ & 49 \\ \hline 146 & 876 \\ & 876 \\ \hline & 0 \end{array}$$

$\sqrt{5776} = 76$

Example 23 : Find the square root of the following by division method :

(*i*) 16384 (*ii*) 71289

Solution : (*i*)

$$\begin{array}{r|l} & 128 \\ \hline 1 & \overline{1}\,\overline{63}\,\overline{84} \\ & 1 \\ \hline 22 & 63 \\ & 44 \\ \hline 248 & 1984 \\ & 1984 \\ \hline & 0 \end{array}$$

$\sqrt{16384} = 128$

(*ii*)

$$\begin{array}{r|l} & 267 \\ \hline 2 & \overline{7}\,\overline{12}\,\overline{89} \\ & 4 \\ \hline 46 & 312 \\ & 276 \\ \hline 527 & 36\,89 \\ & 36\,89 \\ \hline & 0 \end{array}$$

$\sqrt{71289} = 267$

EXERCISE 3.5

Use Cordova Smart Class Software on the smart board in class to do Exercise.

Find the square root of the following by division method :

1. 9801	2. 54756	3. 8649	4. 4489
5. 3249	6. 10201	7. 12100	8. 27225
9. 20449	10. 29241	11. 26569	12. 57121

Word Problems

Example 24 : Find the least number which must be subtracted from 4216 so as to get a perfect square. Also, find the square root of this perfect square number.

Solution : Let us first find the square root of 4216.

```
       64
   6 | 4216
     | 36
 124 |  616
     |  496
     |  120
```

It implies that $(64)^2$ is less than 4216 by 120. So, in order to get a perfect square we have to subtract 120 from 4216.

∴ The required perfect square number is $4216 - 120 = 4096$

We now find the square root of 4096.

```
       64
   6 | 4096
     | 36
 124 |  496
     |  496
     |    0
```

We get, $\sqrt{4096} = 64$

Example 25 : Find the least number which must be added to 1500 so as to get a perfect square number. Also find the square root of this perfect square number.

Solution : Let us find the square root of 1500 by division method.

```
      38
  3 | 1500
    | 9
 68 |  600
    |  544
    |   56
```

It implies $(38)^2 < 1500$.

Now, the next number is 39.

$(39)^2 = 39 \times 39 = 1521$

Also, $(39)^2 - 1500 = 1521 - 1500 = 21$

Hence, the number to be added is 21.

Square number $= 1500 + 21 = 1521$

Now, we find the square root of 1521.

```
      39
  3 | 1521
    | 9
 69 |  621
    |  621
    |    0
```

The square root of 1521 is 39.

Example 26 : Find the greatest four digit number which is a perfect square.

Solution : The greatest four digit number is 9999.

Let us find the square root of 9999.

```
       99
   9 | 9999
     | 81
 189 | 1899
     | 1701
     |  198
```

From the square root we can notice that

$(99)^2$ is less than 9999 by 198.

If we subtract the remainder 198 from the number, we get a perfect square number.

$9999 - 198 = 9801$

```
       99
   9 | 9801
     | 81
 189 | 1701
     | 1701
     |    0
```

Hence, 9801 is the greatest four digit number which is a perfect square.

As, $\sqrt{9801} = 99$

Example 27 : Find the smallest six digit number which is a perfect square.

Solution : The smallest six digit number is 100000. Let us find the square root of 100000.

```
       316
   3 | 100000
     | 9
  61 |  100
     |   61
 626 |  3900
     |  3756
     |   144
```

Also, $(317)^2 = 100489$

Now, $(317)^2 - 100000 = 100489 - 100000 = 489$

We notice that $(316)^2 < 100000$. Thus, if we add 489 to 100000, it becomes a perfect square. Hence, the smallest six digit number, which is a perfect square, is $100000 + 489 = 100489$.

To find the number of digits in the square root without calculating the square root

If we observe the square roots of numbers, calculated by division method, the bar placed over the pair of numbers indicates the number of digits the square root of the numbers will have.

Square root number	Number of bars	Solution (square root of number)	No. of digits in the square root
$\sqrt{\overline{64}}$	1	8	1
$\sqrt{\overline{4}\,\overline{41}}$	2	21	2
$\sqrt{\overline{12}\,\overline{96}}$	2	36	2
$\sqrt{\overline{2}\,\overline{56}\,\overline{00}}$	3	160	3

The number of bars is the number of digits in the square root of the given number.

Example 28 : Without calculating the square root, find the number of digits in the square root of the following numbers :

(i) 36864 (ii) 1960000 (iii) 271441

Solution : (i) $\sqrt{36864} = \sqrt{\overline{3}\,\overline{68}\,\overline{64}}$, square root will have 3 digits.

(ii) $\sqrt{1960000} = \sqrt{\overline{1}\,\overline{96}\,\overline{00}\,\overline{00}}$, square root will have 4 digits.

(iii) $\sqrt{271441} = \sqrt{\overline{27}\,\overline{14}\,\overline{41}}$, square root will have 3 digits.

EXERCISE 3.6

Use Cordova Smart Class Software on the smart board in class to do Exercise.

1. Without calculating the square root, find the number of digits in the square root of the following numbers :
   (i) 36864 (ii) 28900 (iii) 106276 (iv) 4507129
   (v) 32400 (vi) 5625 (vii) 24336
2. Find the greatest 5-digit number which is a perfect square.
3. Find the least number of four digits which is a perfect square.

4. Find the least number which must be subtracted from each of the following numbers to make it a perfect square :

(i) 5045 (ii) 18265 (iii) 194491 (iv) 26535

5. Find the least number which must be added to each of the following numbers to make it a perfect square.

(i) 3720 (ii) 115580 (iii) 4931

To find the square root of rational numbers (fractions)

If a and b are two natural numbers, then $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

If the given fraction is a mixed fraction, it is converted into an improper fraction. To find the square root of the rational number $\frac{a}{b}$, we find the square root of the numerator and the square root of the denominator separately.

Example 29 : Find the square root of :

(i) $\frac{196}{225}$ (ii) $9\frac{67}{121}$

Solution : (i) Let $\frac{a}{b} = \frac{196}{225}$

$a = 196,\ b = 225$

$$\begin{array}{r|l} & 14 \\ \hline 1 & \overline{1}\,\overline{96} \\ & 1 \\ \hline 24 & 96 \\ & 96 \\ \hline & 0 \end{array} \qquad \begin{array}{r|l} & 15 \\ \hline 1 & \overline{2}\,\overline{25} \\ & 1 \\ \hline 25 & 125 \\ & 125 \\ \hline & 0 \end{array}$$

Now, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{196}}{\sqrt{225}} = \frac{14}{15}$

(ii) Let $\frac{a}{b} = 9\frac{67}{121} = \frac{1156}{121},\ \sqrt{\frac{a}{b}} = \sqrt{\frac{1156}{121}} = \frac{\sqrt{1156}}{\sqrt{121}}$

$$\begin{array}{r|l} & 34 \\ \hline 3 & \overline{11}\,\overline{56} \\ & 9 \\ \hline 64 & 256 \\ & 256 \\ \hline & 0 \end{array} \qquad \begin{array}{r|l} & 11 \\ \hline 1 & \overline{1}\,\overline{21} \\ & 1 \\ \hline 21 & 21 \\ & 21 \\ \hline & 0 \end{array}$$

$\sqrt{a} = \sqrt{1156} = 34,$ $\sqrt{b} = \sqrt{121} = 11$

Hence, $\sqrt{\frac{1156}{121}} = \frac{34}{11} = 3\frac{1}{11}$

Example 30 : Find the value of $\sqrt{\frac{1875}{3888}}$.

Solution : The given fraction is $\frac{1875}{3888}$.

On simplifying, $\frac{1875}{3888} = \frac{625}{1296}$

$$\sqrt{\frac{1875}{3888}} = \sqrt{\frac{625}{1296}} = \frac{\sqrt{625}}{\sqrt{1296}}$$

Puzzle

A school has a strange maths teacher. On the first day, he has his students of class VIII perform an odd opening day ceremony.
There are one hundred lockers in the school and one hundred students in that class.
The teacher asks the first student to go to every locker and open it. Then he asks the second student to go to every second locker and close it. Then he asks the third student to go to every third locker and, if it is closed to open it, and if it is open to close it. Then he asks the fourth student to go to every fourth locker and, if it is closed to open it, and if it is open to close it, and so on.
After the process is completed with the hundredth student, how many lockers are open?

$$\begin{array}{r|l} & 25 \\ \hline 2 & \overline{6}\,\overline{25} \\ & 4 \\ \hline 45 & 225 \\ & 225 \\ \hline & 0 \end{array} \qquad \begin{array}{r|l} & 36 \\ \hline 3 & \overline{12}\,\overline{96} \\ & 9 \\ \hline 66 & 396 \\ & 396 \\ \hline & 0 \end{array}$$

Hence, $\sqrt{\frac{1875}{3888}} = \sqrt{\frac{625}{1296}} = \frac{25}{36}$

EXERCISE 3.7

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Simplify the following and write in the form of $\frac{p}{q}$.

(i) $\sqrt{\frac{16}{25}}$ (ii) $\sqrt{6\frac{1}{4}}$ (iii) $\sqrt{27\frac{1}{25}}$ (iv) $\sqrt{7\frac{9}{16}}$

2. Simplify :

(i) $\sqrt{\frac{80}{405}}$ (ii) $\sqrt{\frac{1225}{12321}}$ (iii) $\sqrt{1\frac{155}{169}}$ (iv) $\sqrt{6\frac{145}{256}}$

(v) $\sqrt{84\frac{37}{121}}$ (vi) $\sqrt{\frac{1183}{2023}}$ (vii) $\sqrt{80\frac{244}{729}}$ (viii) $\sqrt{75\frac{46}{49}}$

3. The area of a square field is $23\frac{394}{729}$ sq.m. Find its side.

4. The area of a square field is $35\frac{1}{144}$ sq.m. Find its side.

5. Evaluate :

(i) $\frac{\sqrt{441}+\sqrt{169}}{\sqrt{441}-\sqrt{169}}$ (ii) $\frac{\sqrt{576}+\sqrt{196}}{\sqrt{576}-\sqrt{196}}$

Square Root of Decimals

Observe the following examples :

$0.2 \times 0.2 = 0.04$ $\therefore \sqrt{0.04} = 0.2$

$0.3 \times 0.3 = 0.09$ $\therefore \sqrt{0.09} = 0.3$

$1.3 \times 1.3 = 1.69$ $\therefore \sqrt{1.69} = 1.3$

$6.5 \times 6.5 = 42.25$ $\therefore \sqrt{42.25} = 6.5$

$0.51 \times 0.51 = 0.2601$ $\therefore \sqrt{0.2601} = 0.51$

$0.322 \times 0.322 = 0.103684$ $\therefore \sqrt{0.103684} = 0.322$

It is clear from the above examples, that

(i) square of a decimal fraction has twice the number of decimal places in the given number.

(ii) the number of decimal places in the square root of a given decimal fraction is half the number of decimal places in the given number.

To find the square roots of rational numbers which are in decimal form, the following steps should be followed:

Step 1 : Put bars on the integral parts.

Step 2 : In the decimal part, place the bar making pair of digits starting from the first decimal place. Affix a zero on the extreme right of the decimal part to make it even, if necessary.

Step 3 : Find the square root by division method as done earlier and put the decimal point in the square root before bringing down the pair from decimal part of the number.

Example 31 : Find the square root of the following :

(i) 4.41 (ii) 4.6225 (iii) 150.0625

Solution : (i)

Divisor	2.1
2	$\overline{4}.\overline{41}$
	4
41	41
	41
	0

Hence, $\sqrt{4.41} = 2.1$

(ii)

Divisor	2.15
2	$\overline{4}.\overline{62}\overline{25}$
	4
41	62
	41
425	2125
	2125
	0

$\sqrt{4.6225} = 2.15$

(iii)

Divisor	12.25
1	$\overline{1}\overline{50}.\overline{06}\overline{25}$
	1
22	50
	44
242	606
	484
2445	12225
	12225
	0

$\sqrt{150.0625} = 12.25$

Example 32 : Find the square root of the following numbers.

(i) 0.053361 (ii) 0.008649 (iii) 0.9801

Solution : (i)

Divisor	0.231
2	$0.\overline{05}\overline{33}\overline{61}$
	4
43	133
	129
461	461
	461
	0

$\sqrt{0.053361} = 0.231$

(ii)

Divisor	0.093
0	$0.\overline{00}\overline{86}\overline{49}$
	00
9	86
	81
183	549
	549
	0

$\sqrt{0.008649} = 0.093$

(iii)

Divisor	0.99
9	$0.\overline{98}\overline{01}$
	81
189	1701
	1701
	0

$\sqrt{0.9801} = 0.99$

To find the approximate value of square root

The square roots of the numbers, which are not perfect squares, can be obtained by division method up to a certain decimal place. This can be done by adding suitable number of zeros in pairs to the extreme right of the decimal part of the number.

Example 33 : Find the square root of 2, correct to three decimal places.

Solution : The number 2 is not a perfect square. Since, we require the square root of 2, correct to three places of decimals, we will find the square root up to 4 places. For this, we have to add 4 pairs of zeros after decimal point. We will then approximate the value to three places of decimal.

Divisor	1.4142
1	$\overline{2}.\overline{00}\overline{00}\overline{00}\overline{00}$
	1
24	100
	96
281	400
	281
2824	11900
	11296
28282	60400
	56564
	3836

$\therefore$ $\sqrt{2}$ = 1.4142 up to 4 places of decimal

Since, the digit at 4th place is 2 which is less than 5, we can omit it.

Hence, $\sqrt{2}$ = 1.414, correct to three decimal places

Example 34 : Find the square root of the following, correct to three places of decimal :

(i) $\frac{1}{5}$ (ii) 6.4

Solution : (i) $\sqrt{\frac{1}{5}} = \frac{\sqrt{1}}{\sqrt{5}} = \frac{1}{2.236} = \frac{1000}{2236} = 0.447$, correct to three places of decimal

Divisor	2.2360
2	$\overline{5}.\overline{00}\overline{00}\overline{00}\overline{00}$
	4
42	100
	84
443	1600
	1329
4466	27100
	26796
44720	30400

(ii) $\sqrt{6.4}$ = 2.5298 = 2.530, correct to three places of decimal

Divisor	2.5298
2	$\overline{6}.\overline{40}\overline{00}\overline{00}\overline{00}$
	4
45	240
	225
502	1500
	1004
5049	49600
	45441
50588	415900
	404704
	11196

Example 35 : If $\sqrt{2}$ = 1.4142, find the value of $\sqrt{8}$, correct to three places of decimal.

Solution : $\sqrt{8} = \sqrt{\underline{2\times2}\times2} = 2\times\sqrt{2}$

= 2 × 1.4142 (∵ $\sqrt{2}$ = 1.4142)

= 2.8284 up to 4 places of decimal

$\therefore$ $\sqrt{8}$ = 2.828, correct to three places of decimal.

EXERCISE 3.8

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Find the square root of the following numbers :
   (i) 5.0625 (ii) 31.36 (iii) 51.84 (iv) 98.01
   (v) 0.1521 (vi) 497.29
2. Find the square root of the following, correct to three places of decimal :
   (i) 3 (ii) 7 (iii) 11 (iv) 0.9
   (v) $\frac{7}{8}$ (vi) $4\frac{2}{3}$ (vii) 0.021609 (viii) 0.00155236
3. What is the fraction which when multiplied by itself gives 0.1764?
4. What is the fraction which when multiplied by itself gives 0.7225?
5. The area of a square field is 1892.25 sq. m. Find the side of the square field.
6. The area of a square field is 37056.25 sq. m. Find the side of the square field.

SQUARE ROOTS BY USING SQUARE ROOT TABLES

We have learnt to find out the square root by factorisation method and division method. These methods sometimes become time consuming and cumbersome. From the square root tables, we can find out the approximate value of the square roots of numbers correct to three decimal places.

The following table gives the square roots of the numbers between 1 to 99.

x	$\sqrt{x}$	x	$\sqrt{x}$	x	$\sqrt{x}$	x	$\sqrt{x}$
1	1.000	26	5.099	51	7.141	76	8.718
2	1.414	27	5.196	52	7.211	77	8.775
3	1.732	28	5.292	53	7.280	78	8.832
4	2.000	29	5.385	54	7.348	79	8.888
5	2.236	30	5.477	55	7.416	80	8.944
6	2.449	31	5.568	56	7.483	81	9.000
7	2.646	32	5.657	57	7.550	82	9.055
8	2.828	33	5.745	58	7.616	83	9.110
9	3.000	34	5.831	59	7.681	84	9.165
10	3.162	35	5.916	60	7.746	85	9.220
11	3.317	36	6.000	61	7.810	86	9.274
12	3.464	37	6.083	62	7.874	87	9.327
13	3.606	38	6.164	63	7.937	88	9.381
14	3.742	39	6.245	64	8.000	89	9.434
15	3.873	40	6.325	65	8.062	90	9.487
16	4.000	41	6.403	66	8.124	91	9.539
17	4.123	42	6.481	67	8.185	92	9.592
18	4.243	43	6.557	68	8.246	93	9.644
19	4.359	44	6.633	69	8.307.	94	9.695
20	4.472	45	6.708	70	8.367	95	9.747
21	4.583	46	6.782	71	8.426	96	9.798
22	4.690	47	6.856	72	8.485	97	9.849
23	4.796	48	6.928	73	8.544	98	9.899
24	4.899	49	7.000	74	8.602	99	9.950
25	5.000	50	7.071	75	8.660		

Using this table, we can find the square roots of numbers greater than 100 also.

Example 36 : By using the table for square roots, find the value of :

(i) $\sqrt{24}$ (ii) $\sqrt{64\times 53}$ (iii) $\sqrt{\frac{27}{25}}$

Solution : (i) Look at the table, the entry in the column of $\sqrt{24}$ is 4.899.

$\therefore \sqrt{24} = 4.899$

(ii) $\sqrt{64\times 53} = 8\sqrt{53} = 8\times 7.280 = 58.240$ (From the table, $\sqrt{53} = 7.280$)

(iii) $\sqrt{\frac{27}{25}} = \frac{1}{5}\sqrt{27}$

Look at the table, the entry in the column of $\sqrt{27}$ is 5.196.

$\therefore \sqrt{\frac{27}{25}} = \frac{1}{5}\sqrt{27} = \frac{1}{5}(5.196) = 1.0392$

Example 37 : Using square root table, find the value of

(i) $\sqrt{243}$ (ii) $\sqrt{1.62}$ (iii) $\sqrt{20.23}$

Solution : (i) $\sqrt{243} = \sqrt{81\times 3} = 9\sqrt{3} = 9\times 1.732 = 15.588$ (From the table, $\sqrt{3} = 1.732$)

(ii) $\sqrt{1.62} = \sqrt{\frac{162}{100}} = \sqrt{\frac{81\times 2}{100}} = \frac{9}{10}\sqrt{2}$

From the square root table, $\sqrt{2} = 1.414$

$\therefore \sqrt{1.62} = \frac{9}{10}\sqrt{2} = \frac{9}{10}\times 1.414 = \frac{12.726}{10} = 1.2726$

(iii) $\sqrt{20.23} = \sqrt{\frac{2023}{100}} = \sqrt{\frac{289\times 7}{100}} = \frac{17}{10}\sqrt{7}$

From the square root table, $\sqrt{7} = 2.646$

$\therefore \sqrt{20.23} = \frac{17}{10}\sqrt{7} = \frac{17}{10}\times 2.646 = \frac{44.982}{10} = 4.4982$

Example 38 : Using square root table, evaluate $\sqrt{23.47}$.

Solution : For $\sqrt{23.47}$, we find approximate difference between $\sqrt{24}$ and $\sqrt{23}$.

$\sqrt{24} = 4.899$; $\sqrt{23} = 4.796$ (from the table)

$\sqrt{24} - \sqrt{23} = 4.899 - 4.796 = 0.103$

For the difference of $(24 - 23) = 1$, the difference between $\sqrt{24}$ and $\sqrt{23} = 0.103$

For the difference of $23.47 - 23 = 0.47$, the difference between $\sqrt{23.47}$ and $\sqrt{23} = 0.04841$.

$\therefore \sqrt{23.47} = 4.796 + 0.04841 = 4.844$ (approx.)

EXERCISE 3.9

Use Cordova Smart Class Software on the smart board in class to do Exercise.

Using square root table, find the square root of the following (up to 2 decimal places) :

1. 29 2. 47 3. 78 4. 84 5. 1183
6. 405 7. 801 8. 250 9. 378 10. 15.21
11. 21.92 12. 13.14 13. 83.17 14. 59.45 15. 83.45

FACTS TO REMEMBER

- The square of a number is the product of the number with the number itself.
- A number ending with 2, 3, 7 or 8 cannot be a perfect square.
- Square of an odd number is always an odd number.
- Square of an even number is always an even number.
- If a and b are two perfect squares, then $\sqrt{a\times b} = \sqrt{a}\times\sqrt{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.
- A number ending with an odd number of zeros is never a perfect square.
- The square of a proper fraction is smaller than the fraction.
- The square root of a number x is a number which when multiplied by itself gives x.
- To find the square root of a perfect square :
  - (i) Resolve it into prime factors
  - (ii) Make pairs of similar factors
  - (iii) Choosing one number out of every pair, take the product of prime factors.
- To find the square root of a decimal fraction, add zeros to make decimal places even and find the square root putting decimal point in the square root as soon as the integral part is exhausted.

HOTS QUESTIONS

Find a number whose one-fourth multiplied by its one-sixth becomes 486.

ANSWERS

Exercise 3.1

2. (i) 6 (ii) 1 (iii) 9   3. (ii) and (iii)
4. (i) even (ii) odd (iii) even (iv) even 5. (i) 50 (ii) 38
6. (i) 4225 (ii) 9025 (iii) 42025

Exercise 3.2

1. (8, 15, 17) 2. (12, 35, 37) 3. (18, 80, 82) 4. yes

Exercise 3.3

1. (i) 1; 9 (ii) 4; 6 (iii) 3; 7 (iv) 1; 9
2. (i) 12 (ii) 17 (iii) 13
3. (i) 26 (ii) 32 (iii) 165 (iv) 88 (v) 98 (vi) 125 (vii) 625 (viii) 99

Exercise 3.4

1. 900 2. 337, 4 3. 13; 169 4. 63 m
5. 105 6. 7 7. 119; 34
8. (i) 13; 15 (ii) 7; 20 (iii) 5; 23

Exercise 3.5

1. 99 2. 234 3. 93 4. 67
5. 57 6. 101 7. 110 8. 165
9. 143 10. 171 11. 163 12. 239

Exercise 3.6

1. (i) 3 (ii) 3 (iii) 3 (iv) 4 (v) 3 (vi) 2 (vii) 3
2. 99856 3. 1024 4. (i) 4 (ii) 40 (iii) 10 (iv) 291
5. (i) 1 (ii) 20 (iii) 110

Exercise 3.7

1. (i) $\frac{4}{5}$ (ii) $\frac{5}{2}$ (iii) $\frac{26}{5}$ (iv) $\frac{11}{4}$
2. (i) $\frac{4}{9}$ (ii) $\frac{35}{111}$ (iii) $\frac{18}{13}$ (iv) $\frac{41}{16}$ (v) $\frac{101}{11}$ (vi) $\frac{13}{17}$ (vii) $8\frac{26}{27}$ (viii) $8\frac{5}{7}$
3. $4\frac{23}{27}$ m 4. $5\frac{11}{12}$ m 5. (i) $\frac{17}{4}$ (ii) $\frac{19}{5}$

Exercise 3.8

1. (i) 2.25 (ii) 5.6 (iii) 7.2 (iv) 9.9 (v) 0.39 (vi) 22.3
2. (i) 1.732 (ii) 2.646 (iii) 3.317 (iv) 0.949 (v) 0.935 (vi) 2.160 (vii) 0.147 (viii) 0.039
3. $\frac{21}{50}$ 4. $\frac{17}{20}$ 5. 43.5 m 6. 192.5 m

Exercise 3.9

1. 5.39 2. 6.86 3. 8.83 4. 9.17
5. 34.39 6. 20.12 7. 28.30 8. 15.81
9. 19.44 10. 3.91 11. 4.68 12. 3.62
13. 9.12 14. 7.71 15. 9.13

HOTS : 108

MATHS LAB ACTIVITY

Objective : *Learning about nesting of square roots.*

Sriniwas Ramanujan (1887–1920) was a great Indian mathematical genius. He did a lot of independent research in mathematics during his lifetime. His contributions are acknowledged worldwide and used in many fields today.

One of his many interesting discoveries was nesting of square roots.

A nest is when one square root is under another root.

Examples : (a)

$$2 = \sqrt{4}$$
$$= \sqrt{1+3} = \sqrt{1+\sqrt{9}}$$
$$= \sqrt{1+\sqrt{1+8}} = \sqrt{1+\sqrt{1+2\times 4}}$$
$$= \sqrt{1+\sqrt{1+2\sqrt{16}}} = \sqrt{1+\sqrt{1+2\sqrt{1+15}}}$$
$$= \sqrt{1+\sqrt{1+2\sqrt{1+3\times 5}}}$$
$$= \sqrt{1+\sqrt{1+2\sqrt{1+3\sqrt{25}}}}$$
$$= \sqrt{1+\sqrt{1+2\sqrt{1+3\sqrt{1+...}}}}$$

Now complete the nested square roots :

(b)
$$4 = \sqrt{16}$$
$$= \sqrt{6+10} = \sqrt{6+2\times 5}$$

(c)
$$5 = \sqrt{25}$$
$$= \sqrt{7+18} = \sqrt{7+3\times 6}$$

(d)
$$7 = \sqrt{49}$$
$$= \sqrt{9+40} = \sqrt{9+5\times 8}$$