FOR TEACHERS ONLY

STEP BY STEP

Mathematics

6

CORDOVA®

CONTENTS

1 Knowing Our Numbers

Use **Cordova Smart Class Software** on the **smart board** in class to learn about numbers and numeration.

NATURAL NUMBERS

When we begin to count, the numbers 1, 2, 3, 4, 5, ... come naturally. Hence, these are called **natural numbers** or **counting numbers**. The first or the smallest natural number is 1. There is no largest natural number. There are infinitely many natural numbers.

WHOLE NUMBERS

The natural numbers together with 0 are called **whole numbers**.

Thus, 0, 1, 2, 3, 4, ... are whole numbers.

The smallest whole number is '0' and there is no largest whole number.

DIGITS AND NUMERALS

In order to represent any number, we use ten symbols 0, 1, 2, 3, ..., 9. These ten symbols are called **digits**.

A group of digits denoting a number is called a **numeral**.

For example : 2376, 968541, 6572431 are numerals.

INDIAN SYSTEM OF NUMERATION

Expressing a number in words is called ***numeration*** and representing a number in digits is called ***notation***. In this system, a number is split into groups called ***periods***. Starting from the right, the groups are called ones, thousands, lakhs, crores and arabs.

The ones period contains hundreds, tens and ones places.

The thousands period contains thousands and ten thousands places.

The following place value chart shows the periods and places in Indian system.

Periods	....		Arabs		Crores		Lakhs		Thousands		Ones or Units		
Places			TA Ten arabs	A Arabs	TC Ten crores	C Crores	TL Ten lakhs	L Lakhs	TTh Ten thousands	Th Thousands	H Hundreds	T Tens	O Units or ones
Value			10^{10}	10^{9}	10^{8}	10^{7}	10^{6}	10^{5}	10^{4}	10^{3}	10^{2}	10^{1}	1

We use 'commas' for separating the periods, which help us in reading and writing large numbers. The first comma comes after three digits from the right (*i.e.*, after ones period). The second comma comes two digits later (*i.e.*, after thousands period). The third comma comes after another two digits (*i.e.*, after lakhs period) and so on.

Place Value and Face Value

Let us consider the number 6059.

As the digit 6 occupies thousands place in 6059, the place value of the digit 6 is 6 × 1000 = 6000.

Similarly, the place value of the digit 0 is 0 × 100 = 0;
the place value of the digit 5 is 5 × 10 = 50;
the place value of the digit 9 is 9 × 1 = 9.

Thus, the *place value* of a non-zero digit in a number depends upon the place it occupies in the given number. The place value of digit 0 is always zero regardless of the place it occupies in the given number.

The *face value* of a digit in any number is the digit itself regardless of the place it occupies in the number.

Thus, the place value of a digit in a number is equal to the product of its value and its place in the number.

Expanded Notation

A number can be written in expanded notation, using place values of all its digits.

Consider the number 25027. Each digit has a specific value.

$$25027 = 20000 + 5000 + 0 + 20 + 7$$
$$= 2 \times 10000 + 5 \times 1000 + 0 \times 100 + 2 \times 10 + 7 \times 1$$

A number when written as the sum of the place values of its digits is said to be in its ***expanded notation***.

INTERNATIONAL SYSTEM OF NUMERATION

In the International system of numeration, the periods are ones, thousands, millions and billions.

The following place value chart shows the periods and places in International system.

Periods	....			Billions			Millions			Thousands			Ones		
Places				Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
Values				10^{11}	10^{10}	10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	1

Each period has three places. Commas are used after every three digits from the right.

Example 1 : Write the numeral for 'Ninety-five thousand two hundred seventy-six.'

Solution :

Thousands period		Ones period		
Ten thousands	Thousands	Hundreds	Tens	Ones
9	5	2	7	6

Thus, the number is 95,276.

Example 2 : Write in numerals : Six arab fifty-three crore seventy-two lakh thirty-nine thousand eight.

Solution :

A	TC	C	TL	L	TTh	Th	H	T	O
6	5	3	7	2	3	9	0	0	8

Thus, the numeral is 6,53,72,39,008.

Note : *In a number, if a place is vacant, then put 0 at that place.*

Example 3 : Write 'Seven billion six hundred thirty-five million four hundred ninety-eight thousand seven hundred forty-four' in numerals.

Solution : It represents a collection of 7 billions, 635 millions, 498 thousands, 7 hundreds, 4 tens and 4 ones.

Billions	Millions	Thousands	Ones
7	635	498	744

Thus, the numeral is 7,635,498,744.

Example 4 : Write the number 4367327638 in words in Indian system.

Solution : Inserting commas after each period, the given number can be written as 4,36,73,27,638.

Arabs	Crores	Lakhs	Thousands	Ones
4	36	73	27	638

The given number in words is written as :

Four arab thirty-six crore seventy-three lakh twenty-seven thousand six hundred thirty-eight.

Example 5 : Write each of the following in numerals :

(*i*) Nine million eight hundred eight thousand seven

(*ii*) Seven hundred six million thirteen thousand eighty-six

(*iii*) Five billion thirty-six million thirty-two thousand

Solution : (*i*) Nine million eight hundred eight thousand seven

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
9	8	0	8	0	0	7

Thus, the numeral is 9,808,007.

(*ii*) Seven hundred six million thirteen thousand eighty-six

Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
7	0	6	0	1	3	0	8	6

Thus, the numeral is 706,013,086.

(*iii*) Five billion thirty-six million thirty-two thousand

Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
5	0	3	6	0	3	2	0	0	0

Thus, the numeral is 5,036,032,000.

Example 6 : Write each of the following in expanded notation :

(*i*) 6,28,450 (*ii*) 12,21,307

Solution : (*i*) 6,28,450

$= 6 \times 100000 + 2 \times 10000 + 8 \times 1000 + 4 \times 100 + 5 \times 10 + 0 \times 1$

$= 6,00,000 + 20,000 + 8,000 + 400 + 50$

(*ii*) 12,21,307

$= 1 \times 1000000 + 2 \times 100000 + 2 \times 10000 + 1 \times 1000 + 3 \times 100 + 0 \times 10 + 7 \times 1$

$= 10,00,000 + 2,00,000 + 20,000 + 1,000 + 300 + 7$

Example 7 : Insert commas in the correct positions and write the following numbers in words in Indian and International systems of numeration :

(*i*) 30000009 (*ii*) 2345678 (*iii*) 2020560 (*iv*) 1114232

Solution : (*i*) 30000009

Indian system : 3,00,00,009

Three crore nine

International system : 30,000,009

Thirty million nine

(*ii*) 2345678

Indian system : 23,45,678

Twenty-three lakh forty-five thousand six hundred seventy-eight

International system : 2,345,678

Two million three hundred forty-five thousand six hundred seventy-eight

(*iii*) 2020560

Indian system : 20,20,560

Twenty lakh twenty thousand five hundred sixty

International system : 2,020,560

Two million twenty thousand five hundred sixty

(*iv*) 1114232

Indian system : 11,14,232

Eleven lakh fourteen thousand two hundred thirty-two

International system : 1,114,232

One million one hundred fourteen thousand two hundred thirty-two

Example 8 : What is the place value of 7 in the number 3720861?

Solution :

37,20,861
↑
Lakhs place

As the digit 7 occupies lakhs place, so its place value is 7 × 1,00,000 = 7,00,000

Thus, the place value of 7 in the number 3720861 is 7,00,000.

Example 9 : Find the difference between the place values of two 3s in 935071360.

Solution : The given number is

93, 50, 71, 360
↑ (Crores place) ↑ (Hundreds place)

The place value of 3 at hundreds place = 3 × 100 = 300

The place value of 3 at crores place = 3 × 1,00,00,000 = 3,00,00,000

Required difference = 3,00,00,000 – 300 = 2,99,99,700

Thus, the difference between the place values of two 3s is 2,99,99,700.

Example 10 : Write all the possible 3-digit numbers that can be formed by using the digits 1, 7 and 5 each only once.

Solution : Keeping 1 at ones place, 3-digit numbers formed are 751 and 571.

Keeping 5 at ones place, 3-digit numbers formed are 175 and 715.

Keeping 7 at ones place, 3-digit numbers formed are 157 and 517.

Thus, all possible 3-digit numbers that can be formed are 751, 571, 175, 715, 157 and 517.

Example 11 : How many 3-digit numbers can be formed by the digits 6, 5 and 0 using each digit only once.

Solution : The given digits are 6, 5, 0.

Keeping 0 at ones place, 3-digit numbers formed are 650 and 560.

Keeping 0 at tens place, 3-digit numbers formed are 506 and 605.

The number cannot begin with the digit 0.

Thus, the required 3-digit numbers that can be formed are 650, 560, 506 and 605.

Thus, the number of required 3-digit numbers is four.

EXERCISE 1.1

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Find the place value of the digit 5 in each of the following :

   (*i*) 3572896 (*ii*) 9765321 (*iii*) 79523 (*iv*) 8051

2. Find the product of the place values of two 3s in 6435378.
3. Find the difference of the place value and face value of the digit 5 in 715432.
4. Write each of the following in the expanded form :

   (*i*) 42491 (*ii*) 602732 (*iii*) 20000275

5. How many thousands make a

   (*i*) lakh? (*ii*) crore? (*iii*) million? (*iv*) billion?

6. Write each of the following in words in International system of numeration :

   (*i*) 645007 (*ii*) 2754629 (*iii*) 32604509 (*iv*) 57076342

7. Write each of the following numbers inserting commas in Indian system of numeration. Also, write the numbers in words in Indian system of numeration :

   (*i*) 7904235 (*ii*) 35040027 (*iii*) 2700085 (*iv*) 5974316

8. A certain 9-digit number has only twos in ones period, only threes in thousands period and only fours in millions period. Write the number in words in the Indian system.
9. Keeping the place of 6 in the number 6350947 same, form the smallest number.
10. Write each of the following in numerals :

    (*i*) Five crore forty-two lakh thirty-nine

    (*ii*) Seventy-one lakh seven

    (*iii*) Nine crore twenty-five lakh twelve thousand seven

    (*iv*) Six hundred twenty-one thousand five

    (*v*) Four million forty-nine thousand six hundred eight

    (*vi*) Twenty-nine million four hundred five thousand seven hundred three

11. How many 4-digit numbers in all are there in base 10 system?

    (**Hint :** *Base 10 system refers to decimal number system.*)

12. Form the greatest 6-digit number using the digits 1, 4, 3, 5, 0 and 7 each only once.
13. Write the number corresponding to each of the following :

    (*i*) 7000 + 500 + 30 + 9

    (*ii*) 60000 + 3000 + 600 + 7

    (*iii*) 9000000 + 6000 + 300 + 50 + 4

COMPARISON OF NUMBERS

In order to compare numbers, we follow the steps given below.

1. If the number of digits in the given numbers are unequal, then the number having more digits is greater.
2. If the number of digits in the given numbers are equal, then the digits at the highest place are compared. The number having greater digit (at the highest place) is greater.
3. If the digits at the highest place are equal, then the digits at the next highest place are compared. The number having greater digit (at the next highest place) will be greater.

Example 12 : Which is greater : 57349 or 4024683?

Solution : The number of digits in 57349 is 5.

The number of digits in 4024683 is 7.

Thus, 4024683 > 57349.

Example 13 : Which is greater : 3,69,602 or 3,79,682?

Solution : Both the numbers have same number of digits.

Both the numbers have the same digit at the highest place (i.e., leftmost place).

3,69,602 3,79,682

Since, 6 < 7 (at ten thousands place)

Therefore, 3,79,682 > 3,69,602.

SUCCESSOR AND PREDECESSOR

The successor is one more than the given number whereas predecessor is one less than the given number.

Example 14 : Write the successor of each of the following numbers :

(*i*) 4256 (*ii*) 6320

Solution : (*i*) The successor of 4256 = 4256 + 1 = 4257

(*ii*) The successor of 6320 = 6320 + 1 = 6321

Example 15 : Write the predecessor of each of the following numbers :

(*i*) 2389 (*ii*) 73200

Solution : (*i*) The predecessor of 2389 = 2389 − 1 = 2388

(*ii*) The predecessor of 73200 = 73200 − 1 = 73199

EXERCISE 1.2

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. (*i*) Write down the three consecutive numbers succeeding 70259.
   (*ii*) Write down the three consecutive numbers preceding 37201.
2. Arrange the following numbers in ascending order :
   (*i*) 2456890, 274580, 65432913, 485673 (*ii*) 1606080, 1660080, 234506, 325094
3. Arrange the following numbers in descending order :
   (*i*) 4569235, 569641, 5690084, 4572340 (*ii*) 1023429, 1024567, 1014982, 204561
4. Which is greater : 572845 or 569784?
5. Find the greatest and smallest numbers from each set:
   (*i*) 4356, 4982, 4270, 4451 (*ii*) 26523, 26252, 26170, 26275
   (*iii*) 12560, 12425, 13436, 13529 (*iv*) 8695, 68245, 68435, 68354

LARGE NUMBERS IN PRACTICE

In earlier classes, we have learnt about units for measuring length, mass and capacity. Length is measured in centimetres but it is very small unit to measure the length of a classroom. We use metres for the same. Similarly, we buy items like rice, wheat, sugar in kilograms from grocery shop. To weigh lighter things, we use grams and for much lighter things, we use milligrams. We buy milk in litres and medicines in millilitres.

We know that, 1 metre (m) = 100 centimetres (cm)
1 kilogram (kg) = 1000 grams (g)
1 litre (*l*) = 1000 millilitres (m*l*)

To measure larger things, we use units that are 10 times, 100 times or 1000 times the standard units. We add '**deca**', '**hecto**' or '**kilo**' before the name of the standard unit. To measure smaller things, we use units that are one-tenth, one-hundredth or one-thousandth of the standard unit. To name these units, we add '**deci**', '**centi**' or '**milli**' before the name of the standard unit.

The system of measurement is called **metric system**. Let us arrange the units of length, mass and capacity from smallest to the largest.

Units of Length :

10 millimetres (mm)	=	1 centimetre
10 centimetres (cm)	=	1 decimetre
10 decimetres (dm)	=	1 metre
10 metres (m)	=	1 decametre
10 decametres (dam)	=	1 hectometre
10 hectometres (hm)	=	1 kilometre

Units of Weight :

10 milligrams (mg)	=	1 centigram
10 centigrams (cg)	=	1 decigram
10 decigrams (dg)	=	1 gram
10 grams (g)	=	1 decagram
10 decagrams (dag)	=	1 hectogram
10 hectograms (hg)	=	1 kilogram

Units of Capacity :

10 millilitres (m*l*)	=	1 centilitre
10 centilitres (c*l*)	=	1 decilitre
10 decilitres (d*l*)	=	1 litre
10 litres (*l*)	=	1 decalitre
10 decalitres (da*l*)	=	1 hectolitre
10 hectolitres (h*l*)	=	1 kilolitre (k*l*)

Example 16 : To stitch a shirt, 2 m 15 cm cloth is needed. Out of 40 m cloth, how many shirts can be stitched and how much cloth will be left?

Solution : Cloth needed for a shirt = 2 m 15 cm = {(2 × 100) + 15} cm = 215 cm

Total length of cloth = 40 m = (40 × 100) cm = 4000 cm

Number of shirts that can be stitched = $\frac{4000}{215}$ = 18 (remainder 130)

$$\begin{array}{r} 18 \\ 215\overline{)4000} \\ -215 \\ \hline 1850 \\ -1720 \\ \hline 130 \end{array}$$

Number of shirts that can be stitched = 18

Remaining cloth = 130 cm = 1 m 30 cm

Thus, 18 shirts can be stitched and 1 m 30 cm cloth will be left.

Example 17 : In an election, the winning candidate registered 6,86,700 votes and his nearest rival secured 4,36,300 votes. By what margin did the winning candidate win the election?

Solution : Number of votes that winning candidate got = 6,86,700

Nearest rival's votes = 4,36,300

Margin by which election was won = 6,86,700 – 4,36,300

= 2,50,400

```
  6 8 6 7 0 0
– 4 3 6 3 0 0
  2 5 0 4 0 0
```

Thus, the winning candidate won the election by 2,50,400 votes.

Example 18 : Saini bookstore sold books worth ₹3,25,645 in the first week of a month and books worth ₹5,01,496 in the second week of the month. How much was the sale for the two weeks together? In which week was the sale greater and by how much?

Solution : Sale of books in first week of the month = ₹3,25,645

Sale of books in second week of the month = ₹5,01,496

Total sale (amount) = ₹(3,25,645 + 5,01,496)

= ₹8,27,141

```
    ①①①
  3 2 5 6 4 5
+ 5 0 1 4 9 6
  8 2 7 1 4 1
```

Since 5,01,496 > 3,25,645, therefore the sale in the second week was greater.

Difference = ₹(5,01,496 – 3,25,645)

= ₹1,75,851

```
  ④⑨⑩⑭
  5 0 1 4 9 6
– 3 2 5 6 4 5
  1 7 5 8 5 1
```

Thus, the sale was greater in second week by ₹1,75,851.

Example 19 : Varsha bought 5 kg 600 g of flour, 2 kg 500 g of rice, 3 kg 500 g of sugar, 250 g of salt and 200 g of tea-leaves. Find the total weight of the items bought by her.

Solution : Weight of flour = 5 kg 600 g = {(5 × 1000) + 600} g

= 5600 g

Weight of rice = 2 kg 500 g = {(2 × 1000) + 500} g

= 2500 g

Weight of sugar = 3 kg 500 g = {(3 × 1000) + 500} g

= 3500 g

Weight of salt = 250 g

Weight of tea-leaves = 200 g

Total weight = (5600 + 2500 + 3500 + 250 + 200) g

= 12050 g

= 12 kg 50 g

```
   ②
   5 6 0 0
   2 5 0 0
   3 5 0 0
     2 5 0
 +   2 0 0
 1 2 0 5 0
```

Thus, the total weight of items bought by Varsha is 12 kg 50 g.

Example 20 : A student multiplied 6245 by 62 instead of 52. How much was his answer greater than the correct answer?

Solution :

```
      6 2 4 5                6 2 4 5
        × 6 2                  × 5 2
    1 2 4 9 0              1 2 4 9 0
+   3 7 4 7 0 0        +   3 1 2 2 5 0
    3 8 7 1 9 0            3 2 4 7 4 0
```

Correct answer = 6245 × 52 = 324740

Student's answer = 6245 × 62 = 387190

Difference = 387190 – 324740 = 62450

Thus, his answer was 62450 greater than the correct answer.

Example 21 : Medicine is packed in boxes, each weighing 5 kg 500 g. How many such boxes can be loaded in a van which cannot carry beyond 1100 kg?

Solution :

Weight of each box = 5 kg 500 g = {(5 × 1000) + 500} g

= 5500 g

Weight that the van can carry = 1100 kg = (1100 × 1000) g

= 11,00,000 g

Number of boxes that can be loaded = $\frac{1100000 \text{ g}}{5500 \text{ g}}$ = 200

Thus, 200 boxes can be loaded in the van.

Example 22 : Radius of the Earth is 6400 km and that of Mars is 4300000 m. Whose radius is bigger and by how much?

Solution :

Radius of the Earth = 6400 km

= 6400 × 1000 m = 6400000 m

Radius of Mars = 4300000 m

We see that, 6400000 > 4300000

Difference = (6400000 – 4300000) m

= 2100000 m

Thus, the radius of the Earth is bigger than Mars by 2100000 m.

Example 23 : A vessel has 4 *l* 500 m*l* of curd. How many glasses, each of 250 m*l* capacity, it can fill?

Solution :

Total quantity of curd = 4 *l* 500 m*l*

= {(4 × 1000) + 500} m*l*

= 4500 m*l*

Capacity of a glass = 250 m*l*

Number of glasses required = $\frac{4500}{250}$ = 18

Thus, 4 *l* 500 m*l* of curd can be filled in 18 glasses.

Example 24 : Sukumar had ₹10,00,000 with him. He purchased a colour TV for ₹16,850, a motorcycle for ₹45,000 and a flat for ₹8,70,000. How much money was left with him?

Solution :

Money spent = ₹16,850 + ₹45,000 + ₹8,70,000

= ₹9,31,850

```
   ①①
    1 6 8 5 0
    4 5 0 0 0
+ 8 7 0 0 0 0
-------------
  9 3 1 8 5 0
```

Money left = ₹1000000 – ₹9,31,850

= ₹68,150

```
 ⓪⑨⑨⑨⑨⑩
 1 0 0 0 0 0 0
-  9 3 1 8 5 0
--------------
   0 6 8 1 5 0
```

Thus, Sukumar was left with ₹68,150.

EXERCISE 1.3

Use Cordova Smart Class Software on the smart board in class to do Exercise.

1. A merchant had ₹80290 with him. He placed an order for purchasing 50 ceiling fans at ₹1200 each. How much money will be left with him after the purchase?
2. The distance between the school and Rahim's house is 1 km 420 m. Everyday he walks both ways. Find the total distance covered by him in 5 days.
3. The number of sheets of paper available for making notebooks is 75,000. Each sheet makes 8 pages of a notebook. Each notebook contains 200 pages. How many notebooks can be made from the paper available?
4. A bottle contains 900 *ml* of oil. What is the total quantity of oil contained in 40 such bottles?
5. If 45 kg 450 g of sugar be equally distributed among 15 persons, how much will each get?
6. In an election, the winning candidate got 7,26,249 votes. His nearest rival got 5,26,351 votes and third candidate got 45,460 votes. 2285 votes were declared invalid. What was the total number of votes polled in that constituency?
7. A factory produces 8565 screws in a day. How many screws will it produce in a year if the factory has 291 working days in the year?
8. Population of a town in the year 2015 was 3,25,243. In the year 2016, it was found to have increased by 51,927. What was the population of the town in the year 2016?
9. A box of medicine tablets contains 2,50,000 tablets each weighing 20 mg. What is the weight of all the tablets in the box in grams and in kilograms?
10. A student multiplied 8245 by 35 instead of 32. By how much was his answer greater than the correct answer?
11. 2 m 20 cm of cloth is needed for one shirt. What length of cloth will be required for 16 such shirts?
12. One copy of newspaper has 14 pages. Everyday 12928 copies are printed. How many total pages are printed everyday?
13. In one state, the number of TV sets sold in the year 2014-15 was 8,47,200. In the year 2015-16, the number of TV sets sold was 9,10,345. In which year were more TV sets sold and by how many?
14. Mohit travels 8 km 425 m towards north, then 13 km 210 m towards south and then 15 km 675 m towards west. Calculate the total distance travelled by him.
15. An engine pumps out 850 litres of water in one minute. How many hours will it take to pump out 13,77,000 litres of water?

ESTIMATION

In some situations, we need only a rough estimate of a number instead of the actual number. Estimation gives us an approximate answer which is very near to the correct answer.

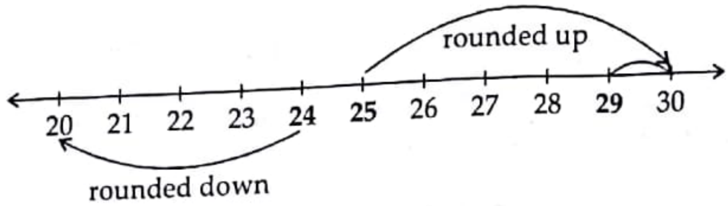
For example :
- The attendance in a cricket match between India and Pakistan was approximately 1,00,000. The figure shows that nearly 1,00,000 people were present during the match.
- Suppose a bus ride takes 27 minutes, we can say the bus ride took around 30 minutes.

Estimating or Rounding off Numbers to the Nearest Tens

Consider the numbers 24 and 29.

If we plot these numbers on the number line, we find that 24 is nearer to 20 than to 30. So, we **rounded down** 24 as 20. Thus, 24 is rounded off to 20 correct to the nearest tens.

29 is nearer to 30 than to 20. So, 29 is **rounded up** to 30. Thus, 29 is rounded off to 30 correct to the nearest tens. If we consider number 25, it is halfway between 20 and 30 on the number line. By convention, we round up 25 to 30.



Example 25 : Round off 3128 to the nearest tens.

Solution : 3128 lies between 3120 and 3130.
It is nearer to 3130 than to 3120. So, 3128 is rounded off as 3130 correct to the nearest tens.

Example 26 : Round off each of the following to the nearest tens :

(i) 93 *(ii)* 754 *(iii)* 997

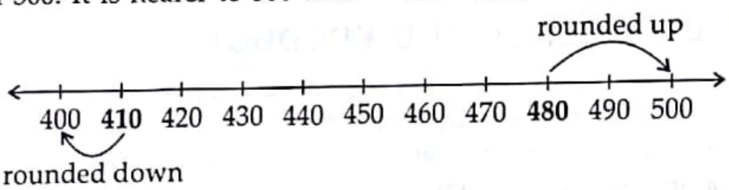
Solution : *(i)* 93 lies between 90 and 100. It is nearer to 90 than to 100. Thus, 93 is rounded off as 90.

(ii) 754 lies between 750 and 760. It is nearer to 750 than to 760. Thus, 754 is rounded off as 750.

(iii) 997 lies between 990 and 1000. It is nearer to 1000 than to 990. Thus, 997 is rounded off as 1000.

Estimating or Rounding off Numbers to the Nearest Hundreds

Consider the numbers 410 and 480.
410 lies between 400 and 500. It is nearer to 400 than to 500. 410 is rounded off as 400.
480 lies between 400 and 500. It is nearer to 500 than to 400. Thus, 480 is rounded off as 500.



Example 27 : Round off 7642 to the nearest hundreds.

Solution : 7642 lies between 7600 and 7700.
It is nearer to 7600 than to 7700. So, 7642 is rounded off as 7600 correct to the nearest hundreds.

Example 28 : Round off each of the following numbers to the nearest hundreds :

(i) 583 *(ii)* 4389 *(iii)* 7523

Solution : *(i)* 583 lies between 500 and 600. It is nearer to 600 than to 500. So, 583 is rounded off as 600.

(ii) 4389 lies between 4300 and 4400. It is nearer to 4400 than to 4300. So, 4389 is rounded off as 4400.

(iii) 7523 lies between 7500 and 7600. It is nearer to 7500 than to 7600. So, 7523 is rounded off as 7500.

Estimating or Rounding off Numbers to the Nearest Thousands

Consider the numbers 6250 and 6501.
6250 lies between 6000 and 7000. It is nearer to 6000 than to 7000. So, 6250 is rounded off as 6000.
6501 lies between 6000 and 7000. It is nearer to 7000 than to 6000.
So, 6501 is rounded off as 7000.

Example 29 : Round off 59326 to the nearest thousands.

Solution : 59,326 lies between 59,000 and 60,000. It is nearer to 59,000 than to 60,000. So, 59326 is rounded off to 59000 correct to the nearest thousands.

Example 30 : Round off each of the following numbers to the nearest thousands :

(*i*) 16328 (*ii*) 15623 (*iii*) 17479

Solution : (*i*) 16328 lies between 16000 and 17000. It is nearer to 16000 than to 17000. So, 16328 is rounded off to the nearest thousands as 16000.

(*ii*) 15623 lies between 15000 and 16000. It is nearer to 16000 than to 15000. So, 15623 is rounded off to the nearest thousands as 16000.

(*iii*) 17479 lies between 17000 and 18000. It is nearer to 17000 than to 18000. So, 17479 is rounded off to the nearest thousands as 17000.

EXERCISE 1.4

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Round off each of the following numbers to the nearest tens :

   (*i*) 82 (*ii*) 884 (*iii*) 997
   (*iv*) 13097 (*v*) 26925 (*vi*) 15802

2. Round off each of the following numbers to the nearest hundreds :

   (*i*) 630 (*ii*) 948 (*iii*) 4783
   (*iv*) 5249 (*v*) 27726 (*vi*) 3124

3. Round off each of the following numbers to the nearest thousands :

   (*i*) 3402 (*ii*) 8603 (*iii*) 4718
   (*iv*) 5823 (*v*) 51342 (*vi*) 26012

4. List all numbers which are rounded off to the nearest tens as 420.

ESTIMATING SUM, DIFFERENCE AND PRODUCT

There are many situations where we need to find the answer quickly. When we go to the market to buy variety of items, we need to estimate the amount needed to buy those items. Suppose, you need to buy a book, a notebook and a pen. You have to make sure that you have an amount more enough to buy these items. For this, we round up the amount and get the sum.

There is no rigid rules when we round off the numbers. The procedure depends on the accuracy required and how quickly the estimate is needed.

Estimating Sum

Example 31 : Estimate the sum : 4390 + 16825

Solution : We round off to the nearest thousands.

16825 is rounded off to 17,000 and 4390 is rounded off to 4,000.

Estimated sum = 17,000 + 4,000 = 21,000

Actual sum = 16825 + 4390 = 21215

The estimated sum is very close to the actual sum. Hence, estimation is reasonable.

Example 32 : Estimate each of the following :

(*i*) 624 + 992 (*ii*) 13805 + 2892 (*iii*) 5312 + 439 + 279

Solution : (*i*) 624 and 992 both are 3-digit numbers.

Rounding off to the nearest hundreds,

624 is rounded off as 600.

992 is rounded off as 1000.

Estimated sum = 600 + 1000 = 1600

(ii) The numbers to be added are 13805 and 2892.
Rounding off to the nearest thousands,
13805 is rounded off as 14000.
2892 is rounded off as 3000.
Estimated sum = 14000 + 3000 = 17000

(iii) Rounding off 5312 to the nearest hundreds = 5300
Rounding off 439 to the nearest hundreds = 400
Rounding off 279 to the nearest hundreds = 300
Estimated sum = 5300 + 400 + 300 = 6000

Estimating Difference

Example 33 : Estimate each of the following :

(i) 756 – 378 (ii) 804 – 537 (iii) 47932 – 4727

Solution : (i) Rounding off to the nearest hundreds,
756 is rounded off as 800.
378 is rounded off as 400.
Estimated difference = 800 – 400 = 400

(ii) Rounding off to the nearest hundreds,
804 is rounded off as 800.
537 is rounded off as 500.
Estimated difference = 800 – 500 = 300

(iii) Rounding off to the nearest thousands,
47932 is rounded off as 48000.
4727 is rounded off as 5000.
Estimated difference = 48000 – 5000 = 43000

Estimating Product

Example 34 : Estimate the product (927 × 376) by rounding off each factor to its greatest place.

Solution : 927 and 376 both are 3-digit numbers.
So, we round off each number to the nearest hundreds.
927 is rounded off as 900 and 376 is rounded off as 400.
Estimated product = 900 × 400 = 3,60,000

Example 35 : Estimate 81 × 479 by rounding off each factor to its greatest place.

Solution : 81 is rounded off to 80 (rounding off to the nearest tens)
479 is rounded off to 500 (rounding off to the nearest hundreds)
Estimated product = 80 × 500 = 40000

EXERCISE 1.5

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Estimate the sum by rounding off the numbers to the nearest hundreds :
   (*i*) 692 + 237 (*ii*) 207 + 391 (*iii*) 587 + 182 (*iv*) 509 + 89
2. Estimate the difference by rounding off the numbers to the nearest hundreds :
   (*i*) 883 – 466 (*ii*) 781 – 539 (*iii*) 900 – 675 (*iv*) 629 – 485
3. Estimate the following products :
   (*i*) 28 × 67 (*ii*) 79 × 192 (*iii*) 92 × 697
4. Estimate the following by rounding off each number to its greatest place :
   (*i*) 877 × 725 (*ii*) 9 × 792 (*iii*) 87 × 221
5. Estimate the following results :
   (*i*) 6398 + 17928 (*ii*) 6893 – 789 (*iii*) 39 × 311 (*iv*) 968 × 539

EXPANDING BRACKETS

We can use the brackets to simplify the multiplication process.

Examples : (*i*) Multiply : 107 × 5

$$107 \times 5 = (100 + 7) \times 5$$
$$= 500 + 35 = 535$$

(*ii*) Multiply : 104 × 107

$$104 \times 107 = (100 + 4) \times (100 + 7)$$
$$= (100 + 4) \times 100 + (100 + 4) \times 7$$
$$= 10000 + 400 + 700 + 28 = 11128$$

(*iii*) Multiply : 23 × 105

$$23 \times 105 = (20 + 3) \times (100 + 5)$$
$$= (20 + 3) \times 100 + (20 + 3) \times 5$$
$$= 2000 + 300 + 100 + 15 = 2415$$

(*iv*) Multiply : 78 × 65

$$78 \times 65 = (70 + 8) \times (60 + 5)$$
$$= (70 + 8) \times 60 + (70 + 8) \times 5$$
$$= 4200 + 480 + 350 + 40 = 5070$$

ROMAN NUMERALS

We have learnt about the Indian and International systems of numeration. One of the early systems of numeration is developed by the Romans and is still in common use. We can see the Roman numerals on the face of clocks. It is still used in mentioning number of chapters, numbers of issues of magazine, denoting sections of a class, denoting ranks in the class. Roman numbers do not have zero and the concept of place values.

The numbers 1 to 100 are written using Roman numerals in the table given below :

Hindu-Arabic Numeral	Roman Numeral	Hindu-Arabic Numeral	Roman Numeral	Hindu-Arabic Numeral	Roman Numeral	Hindu-Arabic Numeral	Roman Numeral	Hindu-Arabic Numeral	Roman Numeral
1	I	21	XXI	41	XLI	61	LXI	81	LXXXI
2	II	22	XXII	42	XLII	62	LXII	82	LXXXII
3	III	23	XXIII	43	XLIII	63	LXIII	83	LXXXIII
4	IV	24	XXIV	44	XLIV	64	LXIV	84	LXXXIV
5	V	25	XXV	45	XLV	65	LXV	85	LXXXV
6	VI	26	XXVI	46	XLVI	66	LXVI	86	LXXXVI
7	VII	27	XXVII	47	XLVII	67	LXVII	87	LXXXVII
8	VIII	28	XXVIII	48	XLVIII	68	LXVIII	88	LXXXVIII
9	IX	29	XXIX	49	XLIX	69	LXIX	89	LXXXIX
10	X	30	XXX	50	L	70	LXX	90	XC
11	XI	31	XXXI	51	LI	71	LXXI	91	XCI
12	XII	32	XXXII	52	LII	72	LXXII	92	XCII
13	XIII	33	XXXIII	53	LIII	73	LXXIII	93	XCIII
14	XIV	34	XXXIV	54	LIV	74	LXXIV	94	XCIV
15	XV	35	XXXV	55	LV	75	LXXV	95	XCV
16	XVI	36	XXXVI	56	LVI	76	LXXVI	96	XCVI
17	XVII	37	XXXVII	57	LVII	77	LXXVII	97	XCVII
18	XVIII	38	XXXVIII	58	LVIII	78	LXXVIII	98	XCVIII
19	XIX	39	XXXIX	59	LIX	79	LXXIX	99	XCIX
20	XX	40	XL	60	LX	80	LXXX	100	C

All Roman numerals are formed by using the seven letters – I, V, X, L, C, D and M in different combinations following certain rules.

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

Rules for Writing Roman Numerals

Rule 1 : *If a symbol is repeated, the number represented by them is their sum.*

II = 1 + 1 = 2, III = 1 + 1 + 1 = 3,

XX = 10 + 10 = 20, XXX = 10 + 10 + 10 = 30

Remember :
- *No Roman numeral is repeated more than thrice.*
- *V, L and D are never repeated.*

Rule 2 : *If a symbol of smaller value is written to the right of a symbol of greater value, the value of resulting numeral is equal to the sum of the values of the symbols.*

VI = 5 + 1 = 6, XXV = 10 + 10 + 5 = 25

XV = 10 + 5 = 15, XVIII = 10 + 5 + 1 + 1 + 1 = 18

Rule 3 : *If a symbol of smaller value is written to the left (before) of a symbol of greater value, the resulting numeral is equal to the difference of the numerals.*

IV = 5 – 1 = 4, IX = 10 – 1 = 9,

XL = 50 – 10 = 40 XC = 100 – 10 = 90.

Rule 4 : *If a symbol of smaller value is placed between two symbols of greater values, it is always subtracted from the symbol of greater value to its right.*

XIV = 10 + 5 – 1 = 14,   XIX = 10 + 10 – 1 = 19,

MCD = 1000 + 500 – 100 = 1400   CXIV = 100 + 10 + (5 – 1) = 114

Remember :
- *V, L and D are never written to the left of a numeral of greater value.*
- *I can be subtracted from V and X only.*
- *X can be subtracted from L and C only.*
- *C can be subtracted from D and M only.*

Example 36 : Write the Roman numerals for each of the following :

(*i*) 98   (*ii*) 69   (*iii*) 586   (*iv*) 444

(*v*) 527   (*vi*) 468   (*vii*) 356   (*viii*) 140

Solution :

(*i*) 98 = 90 + 8 = (100 – 10) + 8 = XC + VIII = XCVIII

(*ii*) 69 = 50 + 10 + 9 = L + X + IX = LXIX

(*iii*) 586 = 500 + 50 + 30 + 5 + 1 = D + L + XXX + V + I = DLXXXVI

(*iv*) 444 = (500 – 100) + (50 – 10) + 4 = CD + XL + IV = CDXLIV

(*v*) 527 = 500 + 20 + 7 = D + XX + VII = DXXVII

(*vi*) 468 = (500 – 100) + (50 + 10) + 8 = CD + LX + VIII = CDLXVIII

(*vii*) 356 = 300 + 50 + 6 = CCC + L + VI = CCCLVI

(*viii*) 140 = 100 + (50 – 10) = C + XL = CXL

Example 37 : Write the following in Hindu-Arabic numerals :

(*i*) IX   (*ii*) XXXIV   (*iii*) XL   (*iv*) LXV

(*v*) XC   (*vi*) DCL   (*vii*) CLXV   (*viii*) DCXLI

Solution :

(*i*) IX = 9

(*ii*) XXXIV = XXX + IV = 30 + 4 = 34

(*iii*) XL = (50 – 10) = 40

(*iv*) LXV = L + X + V = 50 + 10 + 5 = 65

(*v*) XC = 100 – 10 = 90

(*vi*) DCL = D + C + L = 500 + 100 + 50 = 650

(*vii*) CLXV = C + L + X + V = 100 + 50 + 10 + 5 = 165

(*viii*) DCXLI = D + C + XL + I = 500 + 100 + 40 + 1 = 641

Example 38 : Compare the following using >, < or = :

(*i*) XC, XL   (*ii*) LXXXIII, XCV   (*iii*) XL, XXX   (*iv*) CDLXV, CCXLIX

Solution : (*i*)

XC = (100 – 10) = 90

XL = (50 – 10) = 40

Since, 90 > 40

∴ XC > XL

(*ii*) LXXXIII = (50 + 30 + 3) = 83

XCV = (100 – 10 + 5) = 95

Since, 83 < 95

∴ LXXXIII < XCV

(*iii*) XL = (50 – 10) = 40

XXX = 30

Since, 40 > 30

∴ XL > XXX

Laughter Time

Q. What quantity is represented by this?

A. 9 [t(h)ree + t(h)ree + t(h)ree]

Q. A dust storm blows through, now how much do you have?

A. 99 [dirty tree + dirty tree + dirty tree]

(*iv*) CDLXV = (500 – 100) +50 + 10 + 5 = 465
CCXLIX = 200 + 40 + 9 = 249
Since, 465 > 249
∴ CDLXV > CCXLIX

EXERCISE 1.6

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Write the Roman numeral for each of the following :
(*i*) 32 (*ii*) 85 (*iii*) 1600 (*iv*) 42
(*v*) 95 (*vi*) 347 (*vii*) 1672 (*viii*) 1491

2. Write the following in Hindu-Arabic numerals :
(*i*) XXVI (*ii*) XLV (*iii*) CLIX (*iv*) CD
(*v*) XLIV (*vi*) LXIV (*vii*) LVIII (*viii*) CMLXV

3. Compare the following using > or < :
(*i*) CXCVII □ CLXXV (*ii*) XXIV □ XLV (*iii*) LXIX □ LXXII
(*iv*) LX □ XC (*v*) XLIV □ LXIV (*vi*) XCII □ LXV

4. Which of the following are meaningless?
(*i*) VXI (*ii*) IC (*iii*) XL (*iv*) VL
(*v*) LXXIV (*vi*) VVIII (*vii*) XCI (*viii*) MVXVX

5. Give the answer for each of the following in Roman numerals :
(*i*) XC + X (*ii*) XXX – V (*iii*) XCII – XXVII
(*iv*) XLIV + XX (*v*) XCV – LXXV (*vi*) LX – XL

FACTS TO REMEMBER

- The smallest natural number is 1 and there is no last or greatest natural number.
- The successor of a natural number is 1 more than the number.
- The predecessor of a natural number is 1 less than the number.
- The place value of any digit depends upon its position in the number.
- The face value of any digit is the digit itself regardless of its position in the number.
- In the Indian system of numeration, the first three places from the right make ones period, next two places make thousands period and next two places make lakhs period and so on.
- In the International system of numeration, the first three places from the right make ones period, next three places make thousands period and then three places make millions period and so on.
- Roman numerals are formed by using the seven symbols I, V, X, L, C, D and M where I = 1, V = 5, X = 10, L = 50, C = 100, D = 500 and M = 1000.
- A number when written as the sum of the place values of its digits is said to be in its expanded notation.
- If the number of digits in the given numbers is unequal, then the number having more digits is greater.
- If the number of digits in the given numbers is equal, then compare the digits at the highest (*i.e.*, left most) place. The number having greater digit at the highest place is greater.

HOTS QUESTIONS

1. Write the greatest number which when rounded off to the nearest hundreds as 6000.
2. Write the smallest 7-digit number having four different digits.

ANSWERS

Exercise 1.1

1. (i) 500000 (ii) 5000 (iii) 500 (iv) 50 2. 9000000 3. 4995
4. (i) 40000 + 2000 + 400 + 90 + 1
   (ii) 600000 + 2000 + 700 + 30 + 2
   (iii) 20000000 + 200 + 70 + 5
5. (i) 100 (ii) 10,000 (iii) 1000 (iv) 1000000
6. (i) Six hundred forty five thousand and seven.
   (ii) Two million seven hundred fifty four thousand six hundred twenty nine.
   (iii) Thirty two million six hundred four thousand five hundred nine.
   (iv) Fifty seven million seventy six thousand three hundred forty two.
7. (i) 79,04,235; Seventy nine lakh four thousand two hundred thirty five.
   (ii) 3,50,40,027; Three crore fifty lakh forty thousand twenty seven.
   (iii) 27,00,085; Twenty seven lakh eighty five.
   (iv) 59,74,316; Fifty nine lakh seventy four thousand three hundred sixteen.
8. Forty four crore forty three lakh thirty three thousand two hundred twenty two. 9. 6034579
10. (i) 5,42,00,039 (ii) 71,00,007 (iii) 9,25,12,007
    (iv) 621,005 (v) 4,049,608 (vi) 29,405,703
11. 9000 12. 754310
13. (i) 7539 (ii) 63607 (iii) 9006354

Exercise 1.2

1. (i) 70260, 70261, 70262 (ii) 37200, 37199, 37198
2. (i) 274580, 485673, 2456890, 65432913
   (ii) 234506, 325094, 1606080, 1660080
3. (i) 5690084, 4572340, 4569235, 569641
   (ii) 1024567, 1023429, 1014982, 204561
4. 572845
5. (i) 4982, 4270 (ii) 26523, 26170 (iii) 13529, 12425
   (iv) 68435, 8695

Exercise 1.3

1. ₹ 20290	2. 14 km 200 m	3. 3000
4. 36 *l*	5. 3 kg 30 g	6. 13,00,345
7. 24,92,415	8. 3,77,170	9. 5000 g, 5 kg
10. 24735	11. 35 m 20 cm	12. 1,80,992
13. 2015-16, 63145	14. 37 km 310 m	15. 27 hours

Exercise 1.4

1. (i) 80 (ii) 880 (iii) 1000 (iv) 13100 (v) 26930 (vi) 15800
2. (i) 600 (ii) 900 (iii) 4800 (iv) 5200 (v) 27700 (vi) 3100
3. (i) 3000 (ii) 9000 (iii) 5000 (iv) 6000 (v) 51000 (vi) 26000
4. 415, 416, 417, 418, 419, 420, 421, 422, 423, 424.

Exercise 1.5

1. (i) 900 (ii) 600 (iii) 800 (iv) 600
2. (i) 400 (ii) 300 (iii) 200 (iv) 100
3. (i) 2100 (ii) 16000 (iii) 63000
4. (i) 630000 (ii) 8000 (iii) 18000
5. (i) 24000 (ii) 6100 (iii) 12000 (iv) 500000

Exercise 1.6

1. (i) XXXII (ii) LXXXV (iii) MDC (iv) XLII (v) XCV
   (vi) CCCXLVII (vii) MDCLXXII (viii) MCDXCI
2. (i) 26 (ii) 45 (iii) 159 (iv) 400 (v) 44 (vi) 64 (vii) 58 (viii) 965
3. (i) > (ii) < (iii) < (iv) < (v) < (vi) >
4. (i), (ii), (iv), (vi) and (viii)
5. (i) C (ii) XXV (iii) LXV (iv) LXIV (v) XX (vi) XX

HOTS Questions

1. 6099
2. 1000023

Example 1 :

Solution :

Example 2

Solution :

NCERT Exemplar Problems

Example 1 : In a city, polio drops were given to 2,12,583 children on Sunday in March 2008 and to 2,16,813 children in the next month. Find the difference of the number of children getting polio drops in the two months.

Solution : Number of children given polio drops in March = 2,12,583

Number of children given polio drops in April = 2,16,813

Difference = 2,16,813 – 2,12,583

= 4230 children.

Example 2 : A mobile number consists of ten digits. The first four digits of the number are 9, 9, 8 and 7. The last three digits are 3, 5 and 5. The remaining digits are distinct and make the mobile number, the greatest possible number. What are these digits?

Solution : Mobile number consists of 10 digits.

First four digits are 9, 9, 8, 7 and last three digits are 3, 5, 5.

9	9	8	7				3	5	5

Since, the remaining digits are distinct. Therefore, the digits left are 0, 1, 2, 4, 6. To make the number greatest, we have to use the three largest digits in descending order i.e.,

9	9	8	7	6	4	2	3	5	5

Therefore, the digits are 6, 4, 2.

2 Whole Numbers

Use **Cordova Smart Class Software** on the **smart board** in class to learn about whole numbers.

WHOLE NUMBERS

We have already learnt about natural numbers which are also called counting numbers. If we add zero '0' in the collection of natural numbers, we get a new collection of numbers called **whole numbers**. Thus, the natural numbers along with 0 form the collection of whole numbers.

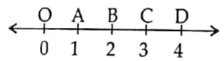
Thus, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, ... are whole numbers.

- The number 0 is the first and smallest whole number.
- All natural numbers are whole numbers.
- There is no last or greatest whole number.
- Adding 1 to a whole number gives us its successor and subtracting 1 from a whole number gives us its predecessor. The whole number 0 has no predecessor.

REPRESENTATION OF WHOLE NUMBERS ON THE NUMBER LINE

The properties of whole numbers can better be understood by representing the whole numbers on the number line. To represent whole numbers on the number line, draw a line and mark any point O on it. It represents 0. Starting from point O, mark points A, B, C, D, ... on the line at equal distances such that OA = 1 unit, AB = 1 unit, BC = 1 unit and so on.

Thus, OA = 1 unit, OB = 2 units, OC = 3 units, OD = 4 units. *i.e.*, A represents 1, B represents 2, C represents 3, D represents 4, ...



Continuing in the same manner, we can represent any whole number.
This line is called the **number line.**

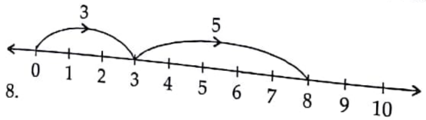
Observing the whole numbers on the number line, we come to the following conclusions :

1. There is no whole number to the left of '0' and every number to its right is greater than it.
2. Every whole number on the number line is less than every whole number to its right. Every whole number on the number line is greater than every whole number to its left.

Addition of Whole Numbers on the Number Line

Example 1 : Represent 3 + 5 on the number line.

Solution : We draw a number line. Starting from 0, we move 3 steps towards right. Then we move 5 steps to the right of 3 to reach at 8.

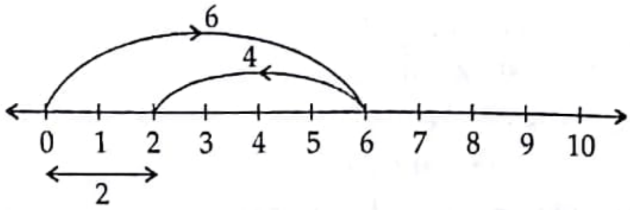


We get, 3 + 5 = 8.

traction of Whole Numbers on the Number Line

nple 2 : Represent 6 – 4 on the number line.

ition : We draw a number line. Starting from 0, we move 6 steps to the right. Now, starting from 6 we move 4 steps to the left to reach at 2.

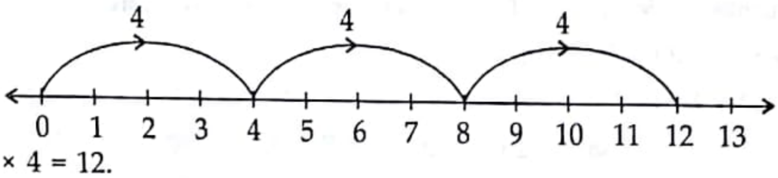


We get, 6 – 4 = 2.

ltiplication of Whole Numbers on the Number Line

ample 3 : Find 3 × 4 using the number line.

lution : We draw a number line. Starting from 0, we move 4 units at a time to the right, making 3 such moves. We reach at 12.



We get, 3 × 4 = 12.

EXERCISE 2.1

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Represent the following numbers on the number line : 2, 3, 5, 7, 11, 14
2. Write the successor of each of the following numbers :
   (*i*) 100909 (*ii*) 34900 (*iii*) 492199
3. Write the predecessor of each of the following numbers :
   (*i*) 1000 (*ii*) 70700 (*iii*) 905000
4. Represent each of the following on the number line.
   (*i*) 3 + 2 (*ii*) 4 + 5 (*iii*) 6 + 2
5. Represent each of the following on the number line.
   (*i*) 7 – 2 (*ii*) 8 – 6 (*iii*) 6 – 2
6. Using the number line, find :
   (*i*) 3 × 3 (*ii*) 3 × 5 (*iii*) 4 × 4

PROPERTIES OF WHOLE NUMBERS

There are four basic operations in Mathematics–addition (+), subtraction (–), multiplication (×) and division (÷). Now, we shall study the specific properties of these operations on whole numbers. These properties help us to understand the numbers better and also make calculations under certain operations simple.

Properties of Addition

1. **Closure property :** The sum of two whole numbers is always a whole number.
   Let a and b be two whole numbers, then $a + b$ is also a whole number.
   Examples : (*i*) 7 + 3 = 10, which is a whole number.
   (*ii*) 6 + 8 = 14, which is a whole number.
   This property is called the closure property. Whole numbers are closed under addition.

2. **Commutative property :** When adding two numbers, it doesn't matter which number is written first, the sum will be the same. If a and b are two whole numbers, then $a + b = b + a$.

Examples : (*i*) $9 + 7 = 7 + 9 = 16$ (*ii*) $12 + 15 = 15 + 12 = 27$

We say that addition is commutative for whole numbers.

3. **Associative property :** The sum of any three whole numbers remains same even if we change the order in which the numbers are grouped.

If a, b, c are any three whole numbers, then $(a + b) + c = a + (b + c)$.

Example : $(5 + 6) + 9 = 5 + (6 + 9) = 20$

This is associativity of addition for whole numbers.

4. **Identity property :** If zero is added to any whole number, the sum is the number itself.

If 'a' is a whole number, then $0 + a = a + 0 = a$.

0 is called the identity for addition of whole numbers (or, ***additive identity***).

Examples : (*i*) $2 + 0 = 0 + 2 = 2$ (*ii*) $101 + 0 = 0 + 101 = 101$

Example 4 : Add the numbers 235, 367 and 128 in as many ways you can.

Solution : The numbers are 235, 367 and 128.

(*i*) $235 + 367 + 128 = 235 + (367 + 128) = 235 + 495 = 730$

(*ii*) $235 + 367 + 128 = 367 + (235 + 128) = 367 + 363 = 730$

(*iii*) $235 + 367 + 128 = 128 + (235 + 367) = 128 + 602 = 730$

Example 5 : Find the sum by suitable rearrangement.

$1 + 2 + 3 + 4 + 896 + 897 + 898 + 899$

Solution : $1 + 2 + 3 + 4 + 896 + 897 + 898 + 899 = (1 + 899) + (2 + 898) + (3 + 897) + (4 + 896)$

$= 900 + 900 + 900 + 900 = 3600$

Example 6 : Add the following and check by reversing the order of addends :

(*i*) $7528 + 2472$ (*ii*) $82567 + 1343$ (*iii*) $19235 + 655$

Solution : (*i*)

$$\begin{array}{r} 7528 \\ +\ 2472 \\ \hline 10000 \\ \hline \end{array} \qquad \begin{array}{r} 2472 \\ +\ 7528 \\ \hline 10000 \\ \hline \end{array}$$

Hence, $7528 + 2472 = 2472 + 7528 = 10000$.

(*ii*)

$$\begin{array}{r} 82567 \\ +\ 1343 \\ \hline 83910 \\ \hline \end{array} \qquad \begin{array}{r} 1343 \\ +\ 82567 \\ \hline 83910 \\ \hline \end{array}$$

Hence, $82567 + 1343 = 1343 + 82567 = 83910$.

(*iii*)

$$\begin{array}{r} 19235 \\ +\ 655 \\ \hline 19890 \\ \hline \end{array} \qquad \begin{array}{r} 655 \\ +\ 19235 \\ \hline 19890 \\ \hline \end{array}$$

Hence, $19235 + 655 = 655 + 19235 = 19890$.

EXERCISE 2.2

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. (*i*) Is the sum of any two even numbers an even number? Verify with taking whole numbers.
   (*ii*) Is the sum of any two odd numbers an odd number? Give example in support of your answer.
2. Determine the sum by suitable rearrangement.
   (*i*) 157 + 376 + 413 + 524 (*ii*) 23 + 446 + 377 + 54 (*iii*) 4001 + 3768 + 2999 + 1232
3. A magic square is an array of numbers and the sum of the numbers in each row, column or diagonal being the same. Fill in the blank cells to complete the magic square.

24		8	15	22
	12	14	21	
11	13	20	27	
17	19	26	28	
18			9	16

4. Add the following and check by reversing the order of addends :
   (*i*) 325 + 4964 (*ii*) 3529 + 741 (*iii*) 973 + 657
5. Find the sum by suitable rearrangement :
   34 + 35 + 36 + 37 + 65 + 63 + 64 + 66

Properties of Subtraction

1. The whole numbers are not closed under subtraction. The difference of two whole numbers is not necessarily a whole number.

   Example : 19 – 12 = 7, which is a whole number.
   7 – 8 is not a whole number.

   If a and b are whole numbers, such that $a > b$ or $a = b$, then $a - b$ is a whole number.
2. The subtraction of whole numbers is not commutative.

   Examples : (*i*) 7 – 3 = 4 but 3 – 7 is not defined.
   (*ii*) 10 – 5 = 5 but 5 – 10 is not defined.

   If a and b are two whole numbers, then $a - b$ is not equal to $b - a$.
3. If 'a' is any whole number, then $a - 0 = a$. But $0 - a$ is not defined.

   Example : 9 – 0 = 9 but 0 – 9 is not defined.
4. If a, b and c are three whole numbers, then $(a - b) - c$ is not equal to $a - (b - c)$.

   For example : Let the numbers be 13, 7 and 5.

   13 – (7 – 5) = 13 – 2 = 11

   and (13 – 7) – 5 = 6 – 5 = 1

   Thus, 13 – (7 – 5) ≠ (13 – 7) – 5.

   Hence, associative property does not hold good for subtraction of whole numbers.
5. If a, b and c are whole numbers such that $a - b = c$, then $b + c = a$.

   For example : We have, 11 – 8 = 3 and 8 + 3 = 11.

Example 7 : Subtract and check the result by corresponding additions.

(*i*) 45697 – 23195 (*ii*) 100010 – 46985

Solution : (*i*)

$$\begin{array}{r} 4\ 5\ 6\ 9\ 7 \\ -\ 2\ 3\ 1\ 9\ 5 \\ \hline 2\ 2\ 5\ 0\ 2 \\ \hline \end{array}$$

Check :

$$\begin{array}{r} 2\ 2\ 5\ 0\ 2 \\ +\ 2\ 3\ 1\ 9\ 5 \\ \hline 4\ 5\ 6\ 9\ 7 \\ \hline \end{array}$$

(*ii*)

$$\begin{array}{r} 1\ 0\ 0\ 0\ 1\ 0 \\ -\ \ 4\ 6\ 9\ 8\ 5 \\ \hline 5\ 3\ 0\ 2\ 5 \\ \hline \end{array}$$

Check :

$$\begin{array}{r} 5\ 3\ 0\ 2\ 5 \\ +\ 4\ 6\ 9\ 8\ 5 \\ \hline 1\ 0\ 0\ 0\ 1\ 0 \\ \hline \end{array}$$

Example 8 : Fill in the correct digits in the following subtraction problems :

(*i*)

$$\begin{array}{r} 8\ 6\ 7\ 5 \\ -\ \square\ \square\ 4\ 9 \\ \hline 4\ 8\ \square\ \square \\ \hline \end{array}$$

(*ii*)

$$\begin{array}{r} 8\ 7\ \square\ 7\ \square \\ -\ \square\ \square\ 6\ 2\ 5 \\ \hline 7\ 3\ 9\ \square\ 1 \\ \hline \end{array}$$

Solution : (*i*)

$$\begin{array}{r} 8\ 6\ 7\ 5 \\ -\ \square\ \square\ 4\ 9 \\ \hline 4\ 8\ \square\ \square \\ \hline \end{array}$$

First we subtract 49 from 75 to get 26.

$$\begin{array}{r} 8\ 6\ 7\ 5 \\ -\ \square\ \square\ 4\ 9 \\ \hline 4\ 8\ 2\ 6 \\ \hline \end{array}$$

We have to find a number which when subtracted from 8675 gives 4826. Such number is the difference of 8675 and 4826.

$$\begin{array}{r} 8\ 6\ 7\ 5 \\ -\ 4\ 8\ 2\ 6 \\ \hline 3\ 8\ 4\ 9 \\ \hline \end{array}$$

Hence,

$$\begin{array}{r} 8\ 6\ 7\ 5 \\ -\ \boxed{3}\ \boxed{8}\ 4\ 9 \\ \hline 4\ 8\ \boxed{2}\ \boxed{6} \\ \hline \end{array}$$

(*ii*)

$$\begin{array}{rl} 8\ 7\ \square\ 7\ \square & \text{— I Row} \\ -\ \square\ \square\ 6\ 2\ 5 & \text{— II Row} \\ \hline 7\ 3\ 9\ \square\ 1 & \\ \hline \end{array}$$

Since, 5 + 1 = 6, the ones digit in row I is 6.

Also, 7 – 2 = 5.

```
   8 7 □ 7 6   ...... I Row
 − □ □ 6 2 5   ...... II Row
   7 3 9 5 1
```

The hundreds digit in first row should be 5 as 15 − 6 = 9.

```
   8 7 5 7 6   ...... I Row
 − □ □ 6 2 5   ...... II Row
   7 3 9 5 1   ...... III Row
```

We have to find a number which when subtracted from 87576 gives 73951.

Such number is the difference of 87576 and 73951.

```
   8 7 5 7 6
 − 7 3 9 5 1
   1 3 6 2 5
```

Hence,

```
   8 7 [5] 7 [6]
 − [1] [3] 6 2 5
   7 3 9 [5] 1
```

Example 9 : What must be added to 135642 to get the sum as the greatest number of six digits?

Solution : The greatest number of six digits is 999999.

The difference of 999999 and 135642 is the required number.

```
   9 9 9 9 9 9
 − 1 3 5 6 4 2
   8 6 4 3 5 7
```

Thus, 864357 should be added to 135642 to get the sum as the greatest number of six digits.

EXERCISE 2.3

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Subtract and check the result by corresponding addition.

```
(i)     3 9 9 9        (ii)     8 5 6 5 9        (iii)     7 4 5 6 5 2
      − 2 6 7 8              − 3 8 9 2 6               − 4 8 2 3 1 0
```

2. What must be added to 8476251 to get the sum as greatest number of seven digits?
3. What is the difference between the largest number of four digits and the smallest number of six digits?
4. Fill in the boxes by using correct digits.

```
(i)     7 6 □ 7 5 7 4        (ii)     5 0 0 0 1 0 6
      −   8 9 □ 9 9 □              −   □ □ 6 7 9 8
        □ 8 0 3 □ 7 5                4 0 2 □ □ □ □
```

5. The digits 4 and 7 of the number 543728 are interchanged. Find the difference between the new number so formed and the original number.
6. The population of a town is 60,000. The number of men is 32,457 and that of women is 13,296. Determine the number of children.
7. What must be added to the least number formed by the digits 0,1,3,5,7 each only once to get the sum as the greatest number formed by the digits 0,2,4,6,8 each only once?
8. Sahil deposited ₹ 3,75,000 in his bank account. Later, he withdrew ₹ 49,265. How much money was left in his account?

Properties of Multiplication

Multiplication is simply repeated addition.

For example : If we have to add 4 five times, we get $4 + 4 + 4 + 4 + 4 = 20$. We can also write it as $5 \times 4 = 20$.

Let us now discuss the various properties of multiplication on whole numbers. We can find the product of large numbers using these properties.

1. **Closure Property :** If a and b are two whole numbers, then their product $a \times b$ is also a whole number.

   Examples : (*i*) $11 \times 9 = 99$, a whole number.

   (*ii*) $12 \times 15 = 180$, a whole number

   Thus, whole numbers are closed under multiplication.

2. **Commutative Property :** If a and b are two whole numbers, then $a \times b = b \times a$.

   Examples : (*i*) $6 \times 7 = 7 \times 6 = 42$

   (*ii*) $13 \times 6 = 6 \times 13 = 78$

   Thus, the product remains the same whatever be the order of numbers to be multiplied.
   Thus, multiplication is commutative for whole numbers.

3. **Associative property :** Let a, b, c be any three whole numbers, then $(a \times b) \times c = a \times (b \times c)$.
   The product of three whole numbers do not change by changing their grouping.

   Examples : (*i*) $(5 \times 6) \times 7 = 30 \times 7 = 210$; $5 \times (6 \times 7) = 5 \times 42 = 210$

   $\therefore$ $(5 \times 6) \times 7 = 5 \times (6 \times 7)$

   (*ii*) $(12 \times 9) \times 5 = 108 \times 5 = 540$; $12 \times (9 \times 5) = 12 \times 45 = 540$

   $\therefore$ $(12 \times 9) \times 5 = 12 \times (9 \times 5)$

   This is associative property for multiplication of whole numbers.

4. **Multiplicative identity :** Let a be a whole number. Then $a \times 1 = 1 \times a = a$.
   The product of any whole number and 1 is the whole number itself.

   Examples : (*i*) $7 \times 1 = 1 \times 7 = 7$ (*ii*) $23 \times 1 = 1 \times 23 = 23$

   1 is called the **multiplicative identity** for whole numbers.

5. **Multiplicative property of zero :** If a is any whole number, then $a \times 0 = 0 \times a = 0$.
   Thus, any whole number multiplied by zero gives zero.

   Examples : (*i*) $9 \times 0 = 0 \times 9 = 0$

   (*ii*) $16 \times 0 = 0 \times 16 = 0$

6. **Distributive Property of multiplication over addition :** If a, b, c are three whole numbers, then $a \times (b + c) = (a \times b) + (a \times c)$ and $(b + c) \times a = (b \times a) + (c \times a)$.

   Examples : (*i*) Consider the expressions $12 \times (7 + 3)$ and $(12 \times 7) + (12 \times 3)$.

   $12 \times (7 + 3) = 12 \times 10 = 120$

   and $(12 \times 7) + (12 \times 3) = 84 + 36 = 120$

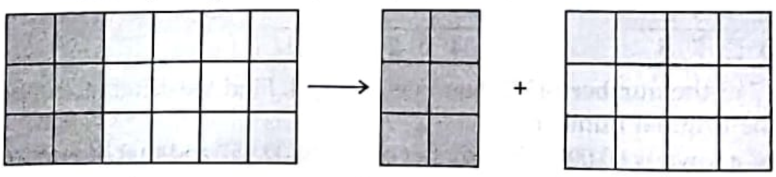
   Thus, $12 \times (7 + 3) = (12 \times 7) + (12 \times 3)$.

   (*ii*) Consider the expressions $(7 + 3) \times 12$ and $(7 \times 12) + (3 \times 12)$.

   $(7 + 3) \times 12 = 10 \times 12 = 120$

   and $(7 \times 12) + (3 \times 12) = 84 + 36 = 120$

   Thus, $(7 + 3) \times 12 = (7 \times 12) + (3 \times 12)$.



3 × 6 squares   3 × 2 squares   3 × 4 squares

We see that $3 \times 6 = (3 \times 2) + (3 \times 4)$

$\Rightarrow$ $3 \times (2 + 4) = (3 \times 2) + (3 \times 4)$

This is known as distributivity of multiplication over addition.

7. **Distributive Property of multiplication over subtraction :** If a, b, c are three whole numbers such that $b > c$, then $a \times (b - c) = a \times b - a \times c$.

Examples : (*i*) Consider the expressions $9 \times (7 - 3)$ and $(9 \times 7) - (9 \times 3)$.

$9 \times (7 - 3) = 9 \times 4 = 36$

and $(9 \times 7) - (9 \times 3) = 63 - 27 = 36$

Thus, $9 \times (7 - 3) = (9 \times 7) - (9 \times 3)$.

(*ii*) Consider the expressions $15 \times (9 - 7)$ and $(15 \times 9) - (15 \times 7)$.

$15 \times (9 - 7) = 15 \times 2 = 30$

and $(15 \times 9) - (15 \times 7) = 135 - 105 = 30$

Thus, $15 \times (9 - 7) = (15 \times 9) - (15 \times 7)$.

This is known as distributivity of multiplication over subtraction.

xample 10 : Find the product by suitable rearrangement :

(*i*) $4 \times 2396 \times 5$ (*ii*) $30921 \times 25 \times 40 \times 2$ (*iii*) $937 \times 2 \times 50$

(*iv*) $8 \times 365 \times 125$ (*v*) $5462 \times 25 \times 4 \times 2$ (*vi*) $4 \times 2 \times 1932 \times 125$

olution : (*i*) $4 \times 2396 \times 5 = 2396 \times (4 \times 5)$ (Associative property)

$= 2396 \times 20 = 47920$

(*ii*) $30921 \times 25 \times 40 \times 2 = (30921 \times 2) \times (25 \times 40)$ (Associative property)

$= 61842 \times 1000 = 61842000$

(*iii*) $937 \times 2 \times 50 = 937 \times (2 \times 50)$ (Associative property)

$= 937 \times 100 = 93700$

(*iv*) $8 \times 365 \times 125 = 365 \times (8 \times 125)$ (Associative property)

$= 365 \times 1000 = 365000$

(*v*) $5462 \times 25 \times 4 \times 2 = (5462 \times 2) \times (25 \times 4)$ (Associative property)

$= 10924 \times 100$

$= 1092400$

(*vi*) $4 \times 2 \times 1932 \times 125 = 1932 \times (4 \times 2 \times 125)$ (Associative property)

$= 1932 \times 1000 = 1932000$

ample 11 : Find the value of each of the following using distributive properties :

(*i*) $(649 \times 8) + (649 \times 2)$ (*ii*) $(6524 \times 69) + (6524 \times 31)$

(*iii*) $(2986 \times 35) + (2986 \times 65)$ (*iv*) $(6001 \times 172) - (6001 \times 72)$

lution : (*i*) $(649 \times 8) + (649 \times 2) = 649 \times (8 + 2)$

(Distributive property of multiplication over addition)

$= 649 \times 10 = 6490$

(*ii*) $(6524 \times 69) + (6524 \times 31) = 6524 \times (69 + 31)$

(Distributive property of multiplication over addition)

$= 6524 \times 100 = 652400$

(*iii*) $(2986 \times 35) + (2986 \times 65) = 2986 \times (35 + 65)$

(Distributive property of multiplication over addition)

$= 2986 \times 100 = 298600$

(*iv*) $(6001 \times 172) - (6001 \times 72) = 6001 \times (172 - 72)$

(Distributive property of multiplication over subtraction)

$= 6001 \times 100 = 600100$

Example 12 : Find each of the following products using properties :

(i) 763 × 92 (ii) 1006 × 273 (iii) 2020 × 48

(iv) 981 × 86 (v) 798 × 42 (vi) 599 × 75

Solution : (i) 763 × 92 = (700 + 60 + 3) × 92

= (700 × 92) + (60 × 92) + (3 × 92)

(Distributive property of multiplication over addition)

= 64400 + 5520 + 276 = 70196

(ii) 1006 × 273 = (1000 + 6) × 273

= (1000 × 273) + (6 × 273)

(Distributive property of multiplication over addition)

= 273000 + 1638 = 274638

(iii) 2020 × 48 = (2000 + 20) × 48

= (2000 × 48) + (20 × 48)

(Distributive property of multiplication over addition)

= 96000 + 960 = 96960

(iv) 981 × 86 = (900 + 80 + 1) × 86

= (900 × 86) + (80 × 86) + (1 × 86)

(Distributive property of multiplication over addition)

= 77400 + 6880 + 86 = 84366

(v) 798 × 42 = (800 – 2) × 42

= (800 × 42) – (2 × 42)

(Distributive property of multiplication over subtraction)

= 33600 – 84 = 33516

(vi) 599 × 75 = (600 – 1) × 75

= (600 × 75) – (1 × 75)

(Distributive property of multiplication over subtraction)

= 45000 – 75 = 44925

Example 13 : Find the values of each using properties.

(i) 3297 × 999 + 3297 (ii) (429 × 10 × 561)– (461 × 4290) (iii) (49 × 34) – (49 × 32)

Solution : (i) 3297 × 999 + 3297 = (3297 × 999) + (3297 × 1)

= 3297 × (999 + 1) (Distributive property of multiplication over addition)

= 3297 × 1000 = 3297000

(ii) (429 × 10 × 561) – (461 × 4290) = (4290 × 561) – (4290 × 461)

= 4290 × (561 – 461)

(Distributive property of multiplication over subtraction)

= 4290 × 100 = 429000

(iii) (49 × 34) – (49 × 32) = 49 × (34 – 32)

(Distributive property of multiplication over subtraction)

= 49 × 2 = 98

Example 14 : The cost of a chair is ₹625. Find the cost of 2725 such chairs.

Solution :

Cost of one chair = ₹625

Cost of 2725 chairs = ₹(625 × 2725)

= ₹625 × (2000 + 700 + 20 + 5)

= ₹[(625 × 2000) + (625 × 700) + (625 × 20) + (625 × 5)]

= ₹(1250000 + 437500 + 12500 + 3125) = ₹17,03,125

Thus, the cost of 2725 chairs is ₹17,03,125.

Example 15 : Find the product of the greatest number of four digits and the greatest number of three digits.

Solution : The greatest number of four digits is 9999.

The greatest number of three digits is 999.

Required product = 9999 × 999

= (10000 – 1) × 999

= 9990000 – 999 (Distributive property of multiplication over subtraction)

= 9989001

Example 16 : The product of two 2-digit numbers is 1081. If the product of their units digits is 21 and that of tens digits is 8, find the numbers.

Solution : The product of units digits = 21 = 3 × 7

Units digits are 3 and 7. (Units digits are 1-digit number)

Now, product of tens digits = 8 = 8 × 1 or 2 × 4

Tens digits are (*i*) 1 and 8 or (*ii*) 2 and 4.

Case I : Two numbers are either 13 and 87 or 17 and 83.

(*i*) 13 × 87 = 13 × (80 + 7)

= (13 × 80) + (13 × 7) (Distributive property of multiplication over addition)

= 1040 + 91 = 1131

(*ii*) 17 × 83 = 17 × (80 + 3)

= (17 × 80) + (17 × 3) (Distributive property of multiplication over addition)

= 1360 + 51 = 1411

Any of these doesn't equal to the required product.

Case II : Two numbers are either 27 and 43 or 23 and 47.

(*i*) 27 × 43 = 27 × (50 – 7)

= (27 × 50) – (27 × 7)(Distributive property of multiplication over subtraction)

= 1350 – 189 = 1161

(*ii*) 23 × 47 = 23 × (50 – 3)

= (23 × 50) – (23 × 3)(Distributive property of multiplication over subtraction)

= 1150 – 69 = 1081

It is given that the product of the numbers is 1081.

Hence, the numbers are 23 and 47.

EXERCISE 2.4

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Find the product of the following by suitable rearrangement :

   (*i*) 4 × 694 × 125 × 2 (*ii*) 5 × 3996 × 20 × 2 (*iii*) 2 × 9897 × 5 (*iv*) 725 × 8 × 50 × 20

2. Find the value of each of the following :

   (*i*) (7909 × 98) + (7909 × 2) (*ii*)(43987 × 45) + (43987 × 55)

   (*iii*) (1297 × 38) + (1297 × 62) (*iv*)(7198 × 197) – (97 × 7198)

3. Find each of the following products using properties :

   (*i*) 7909 × 991 (*ii*) 4980 × 507 (*iii*) 815 × 754

   (*iv*) 3023 × 612 (*v*) 356 × 106

4. Find the product of the largest 3-digit number and the sum of 2456 and 344.

5. In a factory, 3265 toys are made in a day. How many toys will be made in a month of 25 working d
6. A dealer purchased 125 computer sets. If the cost of each is ₹21,346, determine the cost of all computers.
7. Mohit buys 40 chairs and 40 tables. If a chair costs ₹375 and a table costs ₹125, find the total mo spent on chairs and tables.
8. There are 150 pages in a book. Determine the total number of pages in 1652 such books.
9. Subtract the product of 63538 and 35 from the greatest number formed by using the digits 0, 1, 2, 3, 4, each only once.
10. The product of two 2-digit numbers is 1998. If the product of their units digits is 28 and that of te digits is 15, find the numbers.

DIVISION

Division is the inverse operation of multiplication.

Let us divide 32 by 8. Dividing 32 by 8 means finding a whole number which when multiplied by 8 giv 32. Clearly, the number is 4. We write, $32 \div 8 = 4$.

Similarly, $36 \div 9 = 4$ (since $9 \times 4 = 36$)

Division is simply repeated subtraction.

Let us find $18 \div 3$. In order to divide 18 by 3, we subtract 3 repeatedly from 18 till we get 0.

18		
– 3	→	First time
15		
– 3	→	Second time
12		
– 3	→	Third time
9		
– 3	→	Fourth time
6		
– 3	→	Fifth time
3		
– 3	→	Sixth time
0		

When 3 is repeatedly subtracted from 18 six times, we get 0. Therefore, we write $18 \div 3 = 6$.

Note : *If a and b are two whole numbers such that $a < b$, then we cannot subtract b from a. It implies that a smaller whole number cannot be divided by another larger whole number.*

Dividend : The number which is to be divided is called the dividend.
Divisor : The number by which dividend is divided is called the divisor.
Quotient : The number of times the divisor is contained in the dividend is called the quotient.
Remainder : In some divisions, the dividend is not exactly divisible by the divisor. If we divide 14 by 4, we get 2 as left over. The left over number after division is called the remainder.
Division Algorithm : Dividend = Divisor × Quotient + Remainder

where, either remainder = 0 or remainder < divisor

Let us take the properties of division on whole numbers.

1. If a and b are two whole numbers, then $\frac{a}{b}$ is not always a whole number. When a whole number is divided by another whole number, the quotient is not always a whole number. Thus, whole numbers are not closed under division.

For example : $9 \div 3 = 3$ is a whole number.

But $16 \div 3$ is not a whole number as there is no whole number which when multiplied by 3 gives 16.

2. If a and b are two whole numbers, then $a \div b \neq b \div a$.

   For example : $15 \div 5 = 3$ but $5 \div 15 = \frac{1}{3}$

   Thus, division is not commutative.

3. When a whole number is divided by itself, the quotient is 1. When a whole number is divided by 1, the quotient is the whole number itself.

   If $a \neq 0$ and a is a whole number, then $a \div a = 1$ and $a \div 1 = a$.

   For example : $13 \div 13 = 1$ and $13 \div 1 = 13$.

4. When zero is divided by a whole number (other than zero), the quotient is zero.

   If a is a non-zero whole number, then $0 \div a = 0$.

   For example : $0 \div 7 = 0$, $0 \div 11 = 0$.

5. Division by 0 is not defined.

   Let us divide 7 by 0. $\frac{7}{0} = ? \Rightarrow 0 \times ? = 7$

   There is no number that fits here because $0 \times$ any number $= 0$, never 7.

   Let us subtract 0 from 7 repeatedly.

```
  7
 -0    →   First time
 ---
  7
 -0    →   Second time
 ---
  7
 -0    →   Third time
 ---
  7
```

   We never get 0 at any stage of subtraction.

Example 17 : Divide 1968 by 16 and also give its corresponding multiplication fact.

Solution :

```
       123
16 ) 1968
    -16
    ----
      36       Q = 123
     -32       R = 0
     ---
       48
      -48
      ---
        0
```

Multiplication fact : $1968 = 16 \times 123$

Example 18 : Divide 550620 by 532 and check the result.

Solution :

```
          1035
532 ) 550620
     -532
     ----
       1862        Q = 1035
      -1596        R = 0
      -----
        2660
       -2660
       -----
           0
```

Check : $1035 \times 532 = (1000 + 30 + 5) \times 532 = 1000 \times 532 + 30 \times 532 + 5 \times 532$

$= 532000 + 15960 + 2660 = 550620$

Example 19 : What must be multiplied with 493 to get 104516?

Solution : If we divide 104516 by 493, then the quotient will be the required number.

```
        212
493) 104516
     -986
      591
     -493
       986
      -986
         0
```

Q = 212

Thus, 212 must be multiplied with 493 to get 104516.

Example 20 : Find the greatest 5-digit number which is exactly divisible by the greatest 2-digit number.

Solution : The greatest 5-digit number is 99999.

The greatest 2-digit number is 99. Let us divide 99999 by 99.

```
     1010
99) 99999
    -99
      9
     -0
      99
     -99
       09
      -00
        9
```

Q = 1010

R = 9

If we subtract the remainder (*i.e.*, 9) from 99999, we get the required number.

99999 – 9 = 99990

Thus, 99990 is the greatest 5-digit number which is exactly divisible by the greatest 2-digit number.

Example 21 : Find the greatest 7-digit number which is exactly divisible by 729.

Solution : The greatest 7-digit number is 9999999. Let us divide 9999999 by 729.

```
        13717
729) 9999999
     -729
      2709
     -2187
       5229
      -5103
        1269
        -729
         5409
        -5103
          306
```

Q = 13717

R = 306

```
  9 9 9 9 9 9 9
–         3 0 6
  9 9 9 9 6 9 3
```

If we subtract the remainder (*i.e.*, 306) from 9999999, we will get the required number.

Thus, 9999693 is the greatest 7-digit number which is exactly divisible by 729.

Example 22 : 6257 when divided by 43 gives the remainder 22. Find the quotient.

Solution : Dividend = 6257, Divisor = 43, Remainder = 22

We know that :

Dividend = Divisor × Quotient + Remainder

Substituting the values, we get

6257 = 43 × Quotient + 22

$\Rightarrow \quad 43 \times \text{Quotient} = 6257 - 22$

$\Rightarrow \quad 43 \times \text{Quotient} = 6235$

$\Rightarrow \quad \text{Quotient} = \frac{6235}{43} = 145$

Thus, the required quotient is 145.

EXERCISE 2.5

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Divide and check the quotient :
   (*i*) 92619 ÷ 123 (*ii*) 612846 ÷ 582 (*iii*) 75808 ÷ 103
   (*iv*) 607920 ÷ 816 (*v*) 1245616 ÷ 2032 (*vi*) 30400 ÷ 64
2. Find the greatest 7-digit number which is exactly divisible by the greatest 3-digit number.
3. Write the greatest 4-digit number that is exactly divisible by 42.
4. 246 apples can be packed in a carton. How many cartons are required to pack 2755200 apples?
5. *"Division does not distribute over addition."* Give an example to justify the statement.
6. Find the number which when divided by 32 gives 26 as the quotient and 4 as the remainder.
7. Find the number which when divided by 57 gives 39 as the quotient and 7 as the remainder.
8. The product of two numbers is 56088. If one number is 123, what is the other number?
9. On dividing 62345 by 199, the remainder is 58. Find the quotient.
10. On dividing 34567 by 92, the remainder is 67. Find the quotient.

PATTERNS IN WHOLE NUMBERS

Numbers can be represented in shapes using dots. Let • represent 1.

Representing numbers by lines : Every number can be arranged as a line.

2 → • •
3 → • • •
4 → • • • •
5 → • • • • •

Representing numbers as rectangles : Some numbers can be shown as rectangles.

The number 6 can be shown as →

• • •
• • •

There are 2 rows and 3 columns.

or

• •
• •
• •

There are 3 rows and 2 columns.

The number 8 can be shown as →

• • • •
• • • •

There are 2 rows and 4 columns.

or

• •
• •
• •
• •

There are 4 rows and 2 columns.

The numbers which can be shown as a pattern of dots which can be arranged in shape of a rectangle are called rectangular numbers. The numbers 6, 8 are rectangular numbers.

Representing numbers as squares : Some numbers can be shown as squares.

The number 1 can be shown as → (1 × 1)

The number 4 can be shown as → (2 × 2)

The number 9 can be shown as → (3 × 3)

A square number is a number that can be shown as a pattern of dots arranged in the shape of a square. Thus, 1, 4, 9 are square numbers.

Remember : *Every square number is also a rectangular number. But every rectangular number is not a square number.*

Representing numbers as triangles : Some numbers can be shown as triangles.

The number 1 can be shown as → (1)

The number 3 can be shown as → or (1 + 2) = 3

The number 6 can be shown as → or (1 + 2 + 3) = 6

The number 10 can be shown as → or (1 + 2 + 3 + 4) = 10

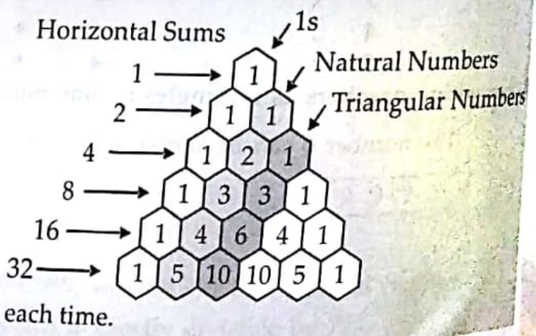
A triangular number is a number that can be shown as a pattern of dots arranged in the shape of a triangle. Thus, 1, 3, 6, 10 are triangular numbers.

Note : *The triangle should have its two sides equal. The number of dots in the rows starting from the bottom row should be like 4, 3, 2, 1. The top row should always have 1 dot.*

Pascal's Triangle

One of the most interesting number patterns is Pascal's triangle (named after Blaise Pascal, a famous French Mathematician).

Each number is equal to the sum of the numbers directly above it.



Observing Patterns

Observation of patterns will guide us in simplifying processes.

Adding 9, 99, 999, ...

118 + 9 = 118 + 10 – 1 = 128 – 1 = 127

118 + 99 = 118 + 100 – 1 = 218 – 1 = 217

1008 + 999 = 1008 + 1000 – 1 = 2008 – 1 = 2007

Subtracting 9, 99, 999, ...

118 – 9 = 118 – 10 + 1 = 108 + 1 = 109

118 – 99 = 118 – 100 + 1 = 18 + 1 = 19

1008 – 999 = 1008 – 1000 + 1 = 8 + 1 = 9

Multiplying by 9, 99, 999, ...

75 × 9 = 75 × (10 – 1) = 750 – 75 = 675

75 × 99 = 75 × (100 – 1) = 7500 – 75 = 7425

75 × 999 = 75 × (1000 – 1) = 75000 – 75 = 74925

Multiplying by 5, 25, 125, ...

$$97 \times 5 = 97 \times \frac{10}{2} = \frac{970}{2} = 485$$

$$97 \times 25 = 97 \times \frac{100}{4} = \frac{9700}{4} = 2425$$

$$97 \times 125 = 97 \times \frac{1000}{8} = \frac{97000}{8} = 12125$$

Example 23 : Given that,

1 + 3 = 2 × 2

1 + 3 + 5 = 3 × 3

1 + 3 + 5 + 7 = 4 × 4

Determine : (*i*) 1 + 3 + 5 + ... + 17 (*ii*) 1 + 3 + 5 + ... + 21.

Solution : It is evident that

sum of first two odd numbers = 2 × 2

sum of first three odd numbers = 3 × 3

sum of first four odd numbers = 4 × 4

(*i*) 1 + 3 + 5 + ... + 17 = Sum of first 9 odd numbers = 9 × 9 = 81

(*ii*) 1 + 3 + 5 + ... + 21 = Sum of first 11 odd numbers = 11 × 11 = 121.

EXERCISE 2.6

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Observe the following pattern and fill in the blanks :

1 × 1	=	1
11 × 11	=	121
111 × 111	=	12321
1111 × 1111	=	....
11111 × 11111	=	....

2. Observe the following pattern and extend it to four more steps :

(7 × 2)	–	3	=	11
(7 × 3)	–	6	=	15
(7 × 4)	–	9	=	19
(7 × 5)	–	12	=	23

3. Observe the following pattern :

$$8 \times 1 + 1 = 9$$
$$8 \times 3 + 1 = 25$$
$$8 \times 6 + 1 = 49$$
$$8 \times 10 + 1 = 81$$

Write three more steps.

4. Study the following pattern and write next three steps :

$$1 \times 8 + 1 = 9$$
$$12 \times 8 + 2 = 98$$
$$123 \times 8 + 3 = 987$$

Puzzle

Using four 4s, mathematical operations +, −, ×, ÷ and brackets, create the numbers 0, 1, 2 and 3.

FACTS TO REMEMBER

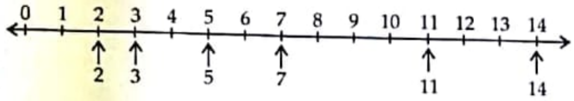
- 0, 1, 2, 3, 4, ... are called whole numbers.
- Every whole number has a successor and every whole number (except zero) has a predecessor.
- Whole numbers can be represented on the number line. We can easily perform operations addition, subtraction and multiplication on the number line. Addition corresponds to moving to the right on the number line and subtraction corresponds to moving to the left. Multiplication corresponds to making jumps of equal distance starting from zero.
- If a, b, c are whole numbers, then
  - (*i*) $a + b$ is a whole number (closure property of addition)
  - (*ii*) $a \times b$ is a whole number (closure property of multiplication)
  - (*iii*) $a - b$ may or may not be a whole number.
  - (*iv*) $a \div b$ may or may not be a whole number.
  - (*v*) $a + b = b + a$ (commutative law of addition)
  - (*vi*) $a \times b = b \times a$ (commutative law of multiplication)
  - (*vii*) $a - b \neq b - a$ (If a and b are unequal)
  - (*viii*) $a \div b \neq b \div a$ (If a and b are unequal)
  - (*ix*) $(a + b) + c = a + (b + c)$ (associative law of addition)
  - (*x*) $(a \times b) \times c = a \times (b \times c)$ (associative law of multiplication)
  - (*xi*) $a \times (b + c) = (a \times b) + (a \times c)$ (distributive law of multiplication over addition)
  - (*xii*) $a \times (b - c) = (a \times b) - (a \times c)$ (If $b > c$, Distributive law of multiplication over subtraction)
  - (*xiii*) $a + 0 = 0 + a = a$ (0 is the additive identity)
  - (*xiv*) $a \times 0 = 0 \times a = 0$ (multiplicative property of zero)
  - (*xv*) $a \times 1 = 1 \times a = a$ (1 is the multiplicative identity)
  - (*xvi*) If a is the dividend, b (where $b \neq 0$) is the divisor, q is the quotient and r is the remainder, then $a = bq + r$ (Division algorithm)
- Division by 0 is not defined.
- Numbers can be represented in shapes using dots.

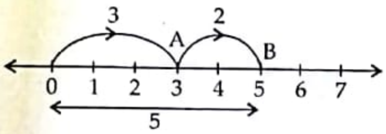
HOTS QUESTIONS

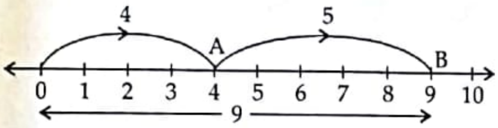
1. ABC and XYZ are each 3-digit numbers, where A, B, C, X, Y and Z are different digits. What is the largest possible value of ABC – XYZ?
2. A kilobyte is equal to 1024 bytes. How many kilobytes and bytes are there in 1,000,000 bytes?

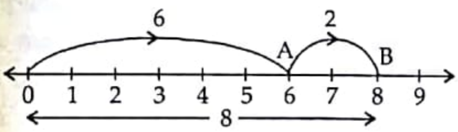
ANSWERS

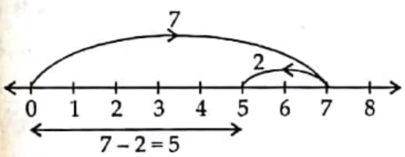
Exercise 2.1

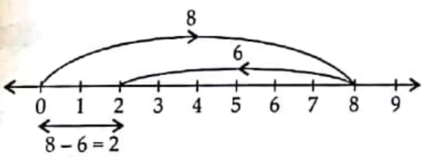
1. 

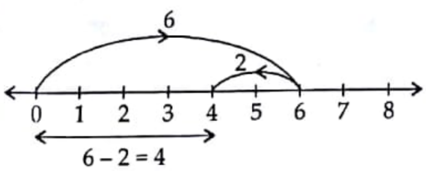
2. (*i*) 100910 (*ii*) 34901 (*iii*) 492200
3. (*i*) 999 (*ii*) 70699 (*iii*) 904999
4. (*i*) 

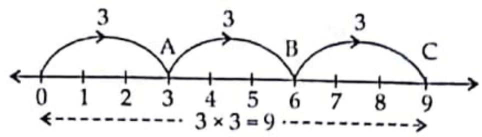
(*ii*) 

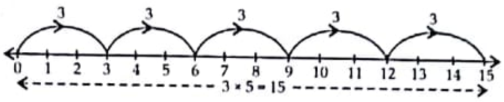
(*iii*) 

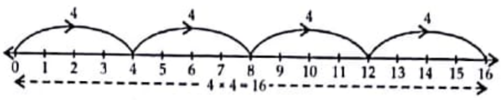
5. (*i*) 

(*ii*) 

(*iii*) 

6. (*i*) 

(*ii*) 

(*iii*) 

Exercise 2.2

1. (*i*) Yes (*ii*) No
2. (*i*) 1470 (*ii*) 900 (*iii*) 12000
3.

24	31	8	15	22
30	12	14	21	23
11	13	20	27	29
17	19	26	28	10
18	25	32	9	16

4. (*i*) 5289 (*ii*) 4270 (*iii*) 1630
5. 400

Exercise 2.3

1. (*i*) 1321 (*ii*) 46733 (*iii*) 263342
2. 1523748
3. 90001
4. (*i*)

$$\begin{array}{r} 7\,6\,9\,7\,5\,7\,4 \\ -\;\;8\,9\,3\,9\,9\,9 \\ \hline 6\,8\,0\,3\,5\,7\,5 \\ \hline \end{array}$$

(*ii*)

$$\begin{array}{r} 5\,0\,0\,0\,1\,0\,6 \\ -\;\;9\,7\,6\,7\,9\,8 \\ \hline 4\,0\,2\,3\,3\,0\,8 \\ \hline \end{array}$$

5. 29700
6. 14247
7. 76063
8. ₹ 325735

Exercise 2.4

1. (*i*) 694000 (*ii*) 799200 (*iii*) 98970 (*iv*) 5800000
2. (*i*) 790900 (*ii*) 4398700 (*iii*) 129700 (*iv*) 719800
3. (*i*) 7837819 (*ii*) 2524860 (*iii*) 614510 (*iv*) 1850076 (*v*) 37736
4. 2797200   5. 81625   6. ₹ 26,68,250   7. ₹ 20000
8. 247800   9. 4319380   10. 37 and 54

Exercise 2.5

1. (*i*) 753 (*ii*) 1053 (*iii*) 736 (*iv*) 745 (*v*) 613 (*vi*) 475
2. 9999990   3. 9996
4. 11200   5. $30 \div (2 + 3) \neq (30 \div 2) + (30 \div 3)$
6. 836   7. 2230   8. 456
9. 313   10. 375

Exercise 2.6

1. 1234321, 123454321
2. $(7 \times 6) - 15 = 27$, $(7 \times 7) - 18 = 31$, $(7 \times 8) - 21$, $(7 \times 9) - 24 = 39$
3. $8 \times 15 + 1 = 121$, $8 \times 21 + 1 = 169$, $8 \times 28 + 1$...
4. $1234 \times 8 + 4 = 9876$, $12345 \times 8 + 5 = 98765$, $123456 \times 8 + 6 = 987654$.

HOTS Questions

1. 885
2. 976 kilobytes 576 bytes

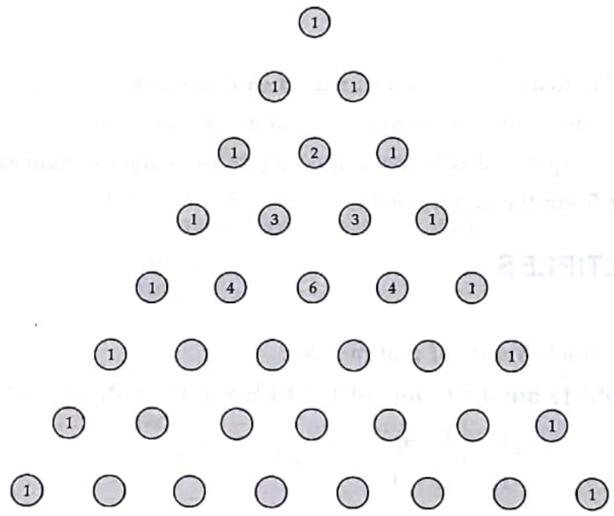
MATHS LAB ACTIVITY

PASCAL'S TRIANGLE

Pascal's triangle is a triangular arrangement of numbers and is named after the French Mathematician, Blaise Pascal.

Observe the triangle carefully.

Each number is equal to the sum of the numbers directly above it.



(*a*) Complete the triangle.

(*b*) How many times do consecutive counting numbers appear?

(*c*) Add the numbers in each row. What number pattern do you observe?

(*d*) Look at the diagonal that begins with 1, 3, 6, ...

Add the first and second numbers, then second and third numbers and so on. Observe the pattern.

(*e*) Look at each row.

1, 11, 121, 1331, ...

Do you observe any pattern?

NCERT Exemplar Problems

Example : The population of a town is 450772. In a survey, it was reported that one out of every 14 persons is illiterate. In all how many illiterate persons are there in the town?

Solution : The population of the town is 450772.

One person is illiterate out of every 14.

Therefore, the total number of illiterate persons = $\frac{450772}{14}$ = 32198

3 Playing with Numbers

Use Cordova Smart Class Software on the smart board in class to learn about factors and multiples.

INTRODUCTION

We know that, 5 × 3 = 15. Mathematically, we can read it in two ways :

(*i*) The product of 5 and 3 is 15 or 15 is a multiple of 3 as well as of 5.

(*ii*) On dividing 15 by 3, the quotient is 5 or on dividing 15 by 5, the quotient is 3.

In other words, 3 and 5 are the factors of 15.

FACTORS AND MULTIPLES

Factors

A *factor* of a number is an exact divisor of that number.

For example : 1, 2, 3, 4, 6 and 12 are the factors of 12 as each of these divides 12 exactly.

12 ÷ 1 = 12; 12 ÷ 2 = 6; 12 ÷ 3 = 4;

12 ÷ 4 = 3; 12 ÷ 6 = 2; 12 ÷ 12 = 1

Properties of Factors

1. *1 is a factor of every number.*

   For example : Factors of 4 are **1**, 2, 4.

   Factors of 6 are **1**, 2, 3, 6.

2. *Every number is a factor of itself.*

   For example : Factors of 8 are 1, 2, 4, **8**.

   Factors of 9 are 1, 3, **9**.

3. *Every factor of a number is less than or equal to the number.*

   For example : The factors of 12 are 1, 2, 3, 4, 6 and 12. Out of these, 12 itself is the largest fact of 12 and all other factors are less than 12.

4. *Every factor of a number is an exact divisor of that number.*

   For example : The factors of 16 are 1, 2, 4, 8 and 16. Each factor of 16 exactly divides 16.

5. *Every number has finite number of factors.*

   Since, every factor of a number is less than or equal to the number (Property 3), therefore, numb of factors of a given number are finite.

Example 1 : Write all the factors of (*i*) 24 (*ii*) 32.

Solution : (*i*) The given number is 24.

$24 = 1 \times 24$; $24 = 2 \times 12$; $24 = 3 \times 8$; $24 = 4 \times 6$

Thus, the factors of 24 are 1, 2, 3, 4, 6, 8, 12, and 24.

(*ii*) The given number is 32.

$32 = 1 \times 32$; $32 = 2 \times 16$; $32 = 4 \times 8$

Thus, the factors of 32 are 1, 2, 4, 8, 16, and 32.

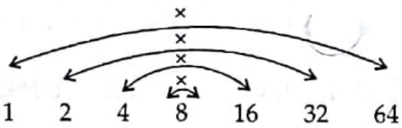
Example 2 : The product of two numbers is 64. Their sum is 20. Find the numbers.

Solution : The product of two numbers is 64. The required two numbers are the factors of 64 such that their sum is 20.

The factors of 64 are 1, 2, 4, 8, 16, 32, and 64.

4 and 16 are two factors whose sum is 20.

Thus, the required numbers are 4 and 16.



Example 3 : Without actual division, show that 11 is a factor of 11011.

Solution :

$$11011 = 11000 + 11 = 11 \times 1000 + 11 \times 1$$
$$= 11 \times (1000 + 1) = 11 \times 1001$$

$\Rightarrow$ 11 is a factor of (11×1001).

Hence, 11 is a factor of 11011.

Multiples

The ***multiples*** of a number are obtained by multiplying it by natural numbers 1, 2, 3, ...

A number is a multiple of each of its factors.

For example : If we multiply 3 by 1, 2, 3, 4, 5, ..., we get 3, 6, 9, 12, 15, 18, 21, ... which are all multiples of 3.

Properties of Multiples

1. *Every number is a multiple of itself.*

   For example : $5 = 5 \times 1$, $6 = 6 \times 1$, $8 = 8 \times 1$, $12 = 12 \times 1$

2. *Every multiple of a number is greater than or equal to the number.*

   For example : The multiples of 6 are 6, 12, 18, 24, 30, ... The smallest multiple is 6 while the rest are greater than 6.

3. *Every number has an infinite number of multiples.*

   The multiples of a given number is obtained by multiplying the number with natural numbers which are infinite. Hence, multiples of a given number are infinite.

Example 4 : Find the multiples of 7 which are greater than 21 but less than 84.

Solution : To find the multiples of 7, we multiply 7 by 1, 2, 3, 4, 5, 6, ...

$7 \times 1 = 7$	$7 \times 4 = 28$,	$7 \times 7 = 49$,	$7 \times 10 = 70$,
$7 \times 2 = 14$	$7 \times 5 = 35$,	$7 \times 8 = 56$,	$7 \times 11 = 77$,
$7 \times 3 = 21$	$7 \times 6 = 42$,	$7 \times 9 = 63$	$7 \times 12 = 84$

Thus, the multiples of 7 greater than 21 and less than 84 are 28, 35, 42, 49, 56, 63, 70, and 77.

Example 5 : Write the first five multiples of 19.

Solution : $19 \times 1 = 19$; $19 \times 2 = 38$; $19 \times 3 = 57$; $19 \times 4 = 76$; $19 \times 5 = 95$.

Thus, the first five multiples of 19 are 19, 38, 57, 76, and 95.

Perfect numbers : If the sum of all the factors of a number is twice the number, then the number is called a *perfect number.*

For example : The factors of 6 are 1, 2, 3, and 6.

Their sum $(1 + 2 + 3 + 6) = 12$, which is 2 times 6. Hence, 6 is a perfect number.

The factors of 28 are 1, 2, 4, 7, 14, and 28.

Their sum (1 + 2 + 4 + 7 + 14 + 28) = 56, which is 2 times 28.

Hence, 28 is also a perfect number.

Even and odd numbers : A natural number which is a multiple of 2 is called an ***even number***. Even nu are exactly divisible by 2. The numbers 2, 4, 6, 8, 10, 12, ... are even numbers.

(*i*) There are infinite even numbers.

(*ii*) An even number ends in 0, 2, 4, 6, or 8.

A natural number which is not a multiple of 2 is called an ***odd number***. Odd numbers are not exactly di by 2. The numbers 1, 3, 5, 7, 9, 11, ... are odd numbers.

(*i*) There are infinite odd numbers.

(*ii*) An odd number ends in 1, 3, 5, 7, or 9.

PRIME AND COMPOSITE NUMBERS

Prime numbers : A natural number which is greater than 1 and whose only factors are 1 and the number is called a ***prime number.***

The numbers 2, 3, 5, 7, 11, 13, 17, ... are prime numbers, each having exactly two factors.

(*i*) There are infinite prime numbers.

(*ii*) 2 is the only prime number which is even. All other prime numbers are odd.

Composite numbers : The numbers having more than two factors are called ***composite numbers.***

For example : Factors of 4 are 1, 2, 4. Factors of 6 are 1, 2, 3, 6. These are composite numbers.

Note : *'1' is neither a prime nor a composite number.*

Sieve of Eratosthenes

We can find prime and composite numbers from 1 to 100 with an easier method. This method was give Greek Mathematician Eratosthenes, in the 3rd century BCE.

List all the numbers from 1 to 100, as shown below (cross out 1 as it is not a prime number).

	2	**3**	~~4~~	**5**	~~6~~	**7**	~~8~~	~~9~~	~~10~~
11	~~12~~	**13**	~~14~~	~~15~~	~~16~~	**17**	~~18~~	**19**	~~20~~
~~21~~	~~22~~	**23**	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	**29**	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	**37**	~~38~~	~~39~~	~~40~~
41	~~42~~	**43**	~~44~~	~~45~~	~~46~~	**47**	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	**53**	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	**59**	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	**67**	~~68~~	~~69~~	~~70~~
71	~~72~~	**73**	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	**79**	~~80~~
~~81~~	~~82~~	**83**	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	**89**	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	**97**	~~98~~	~~99~~	~~100~~

Step 1 :
Step 2 :
Step 3 :
Step 4 :
All the
All the
Thus, th
2, 3, 5,
Perhaps
looked
Co-prin
numbe
For exa
(*i*) A
(*ii*) C
Fo
Twin p
For ex
Prime
triplet
Examp
Soluti
Examp
Soluti
Examp
Soluti
1.
2.
3.
4.
5.

Step 1 : Encircle 2, cross out all the multiples of 2, other than 2 itself.

Step 2 : Next uncrossed number is 3. Encircle 3 and cross out all the multiples of 3, other than 3 itself.

Step 3 : Next uncrossed number is 5. Encircle 5 and cross out all the multiples of 5, other than 5 itself.

Step 4 : Continue this process till all the numbers in the list are either encircled or crossed out.

All the encircled numbers are prime numbers.

All the crossed out numbers are composite numbers.

Thus, the prime numbers from 1 to 100 are as follows :

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

Perhaps Eratosthenes made holes in the paper instead of crossing out the numbers. The paper must have looked like a sieve, hence the name.

Co-prime numbers : The two numbers which have no common factor other than 1 are called ***co-prime numbers***.

***For example* :** The pairs of co-primes are (2, 3), (3, 4), (5, 7), (8, 9).

(*i*) Any two prime numbers are always co-primes.

(*ii*) Co-prime numbers need not be prime numbers.

***For example* :** (9, 10) are co-primes but both 9 and 10 are not prime numbers.

Twin primes : A pair of prime numbers which differ by 2 are called ***twin primes***.

***For example* :** The pairs 3 and 5; 5 and 7; 11 and 13; 17 and 19 are some examples of twin primes.

Prime triplet : A set of three consecutive prime numbers, differing by 2 is called a ***prime triplet***. The only prime triplet is (3, 5, 7).

Example 6 : Can a composite number be odd ? If yes, write the smallest odd composite number.

Solution : Yes, the smallest odd number which has more than two factors is 9.
Thus, 9 is the smallest odd composite number.

Example 7 : Check whether 37 and 39 are twin primes or not.

Solution : 37 and 39 are not twin primes as 39 is not a prime number.

Example 8 : What is the next prime number greater than 97.

Solution : 101 is the next prime number greater than 97 as it is not divisible by 2, 3, 5, 7 or 11.

EXERCISE 3.1

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Which of the following numbers are odd numbers ? Give reasons.

   (*i*) 18 (*ii*) 177 (*iii*) 345

   (*iv*) 390 (*v*) 67 (*vi*) 462

2. Write all the multiples of :

   (*i*) 13 (between 26 and 117) (*ii*) 17 (between 51 and 136)

3. Write all prime numbers between 20 and 35.

4. Check whether the first number is a factor of second number or not.

   (*i*) 13, 65 (*ii*) 15, 60 (*iii*) 17, 64 (*iv*) 8, 72

5. Write each of the following numbers as a sum of two primes :

   (*i*) 5 (*ii*) 12 (*iii*) 36

6. Write down the factors of the following numbers :

(i) 45 (ii) 60 (iii) 78

7. Write all even numbers between 31 and 51.

8. Write all the multiples of 9 less than 100.

9. Express each of the following numbers as the sum of three odd primes :

(i) 21 (ii) 31 (iii) 53

10. Express each of the following numbers as the sum of two odd primes :

(i) 44 (ii) 18 (iii) 24

TESTS FOR DIVISIBILITY OF NUMBERS

Divisibility rules help us to know if a number is divisible by another without actual division.

Divisibility by 2

Even numbers are divisible by 2. They have 0, 2, 4, 6 and 8 in their ones place.

A number is divisible by 2, if its ones digit is 0, 2, 4, 6 or 8.

For example **:** The numbers 1472, 564, 3286, 428, 740 are divisible by 2, since their ones digits are 2, 4, 6, 8 and 0 respectively.

Divisibility by 3

Multiples of 3 are 3, 6, 9, 12, 15, 18, ...

We see that sum of the digits of the multiples of 3 are 3, 6, 9, 1 + 2 = 3, 1 + 5 = 6, 1 + 8 = 9, ...

A number is divisible by 3, if the sum of its digits is divisible by 3.

For example **:** 7563 is divisible by 3 since sum of its digits (7 + 5 + 6 + 3) = 21 is divisible by 3.

Divisibility by 4

A number is divisible by 4, if the last two digits are 0s or if the number formed by its last two digits (i.e., ones and tens) is divisible by 4.

For example **:** The numbers 1800, 2076, 728, 764 are divisible by 4 since the numbers formed by their last two digits are divisible by 4.

Divisibility by 5

The multiples of 5 are 5, 10, 15, 20, 25, 30, ...

Each number has either 0 or 5 in the ones place.

A number is divisible by 5, if its ones digit is either 0 or 5.

For example **:** 6280, 475 are divisible by 5.

Divisibility by 6

We know that, 6 = 2 × 3.

A number is divisible by 6, if it is divisible by 2 and 3 both.

For example **:** The number 43542 is divisible by 2 as its ones digit is 2. It is also divisible by 3 as sum of its digits (4 + 3 + 5 + 4 + 2) = 18 is divisible by 3. Hence, 43542 is also divisible by 6.

Divisibility by 8

A number is divisible by 8, if the number formed by its last three digits is divisible by 8.

For example **:** The number formed by last three digits of 6328 is 328 which is divisible by 8. So, 6328 is also divisible by 8.

Divisibility by 9

The multiples of 9 are 9, 18, 27, 36, ...

Sum of the digits is always 9.

A number is divisible by 9, if the sum of its digits is divisible by 9.

For example **:** The sum of the digits of 36054 is (3 + 6 + 0 + 5 + 4) = 18, which is divisible by 9. Thus, 36054 is also divisible by 9.

Divisibility by 10

A number is divisible by 10, if the digit at its ones place is 0.

For example **:** The numbers 790, 84680 are divisible by 10.

Divisibility by 11

A number is divisible by 11, if the difference between the sum of the digits at odd places (from the right) and the sum of the digits at even places (from the right) is either 0 or a multiple of 11.

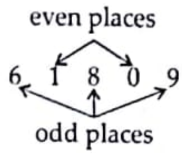
For example **:** Sum of the digits at odd places = 9 + 8 + 6 = 23

Sum of the digits at even places = 0 + 1 = 1

Difference = 23 − 1 = 22

22 is divisible by 11.

Thus, 61809 is divisible by 11.



Example 9 : Which of the following numbers are divisible by 2?

(*i*) 3250 (*ii*) 9628 (*iii*) 7435 (*iv*) 5786

Solution : A number is divisible by 2 if its ones digit is 0, 2, 4, 6 or 8.

(*i*) 3250 is divisible by 2 as its ones digit is 0.

(*ii*) 9628 is divisible by 2 as its ones digit is 8.

(*iii*) 7435 is not divisible by 2 as its ones digit is 5.

(*iv*) 5786 is divisible by 2 as its ones digit is 6.

Example 10: Using divisibility rules, check whether the following numbers are divisible by 3 :

(*i*) 5412 (*ii*) 832 (*iii*) 5463 (*iv*) 519

Solution : A number is divisible by 3 if the sum of its digits is exactly divisible by 3.

(*i*) Since the sum of the digits of 5412 = (5 + 4 + 1 + 2) = 12 is divisible by 3, therefore, 5412 is divisible by 3.

(*ii*) Since the sum of the digits of 832 = (8 + 3 + 2) = 13 is not divisible by 3, therefore, 832 is not divisible by 3.

(*iii*) Since the sum of the digits of 5463 = (5 + 4 + 6 + 3) = 18 is divisible by 3, therefore, 5463 is also divisible by 3.

(*iv*) Since the sum of the digits of 519 = (5 + 1 + 9) = 15 is divisible by 3, therefore, 519 is divisible by 3.

Example 11: Test the divisibility of the following numbers :

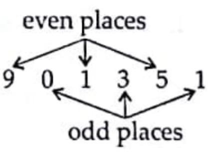
(*i*) 901351 by 11 (*ii*) 136976 by 4 and 8 (*iii*) 57624 by 6.

Solution : (*i*) Sum of the digits at odd places = 1 + 3 + 0 = 4

Sum of the digits at even places = 5 + 1 + 9 = 15

Difference of two sums = 15 − 4 = 11, which is a multiple of 11

Hence, 901351 is divisible by 11.



(*ii*) Since the number formed by the last two digits of 136976 is 76 which is divisible by 4, therefore, 136976 is divisible by 4.

The last three digits of 136976 form a number 976 which is divisible by 8, therefore, 136976 is divisible by 8.

(*iii*) Since 57624 has 4 at ones place, therefore, the number is divisible by 2.
Also, sum of the digits of 57624 = (5 + 7 + 6 + 2 + 4) = 24 is divisible by 3, therefore number is divisible by 3.
Since 57624 is divisible by 2 and 3 both, therefore 57624 is divisible by 6.

Properties of Divisibility

1. *If a number is divisible by another number, it must be divisible by each of the factor of that number.*
   Example : We know that 36 is divisible by 12.
   The factors of 12 are 1, 2, 3, 4, 6, and 12.
   Hence, 36 is divisible by each one of the factors 1, 2, 3, 4, 6, and 12.
2. *If a number is divisible by two or more co-prime numbers, it must be divisible by their product.*
   Example : 80 is divisible by 4 and 5.
   As 4 and 5 are co-prime numbers, so 80 must be divisible by 4 × 5 = 20, which is
3. *If two given numbers are divisible by a number, then their sum is also divisible by that number.*
   Example : 16 and 20 both are divisible by 4. The number (16 + 20) = 36 is also divisible by
4. *If two given numbers are divisible by a number, then their difference is also divisible by that number.*
   Example : 15 and 40 both are divisible by 5. Their difference (40 – 15) = 25 is also divisible

Remember :
- *The product of two consecutive whole numbers is divisible by 2.*
  Example : *3 × 4 = 12 is divisible by 2.*
- *The product of three consecutive numbers is always divisible by 6.*
  Example : *3 × 4 × 5 = 60 is divisible by 6.*
- *The sum of two consecutive odd numbers is divisible by 4.*
  Example : *91 + 93 = 184 is divisible by 4.*

To Find Prime Numbers Between 100 and 400 : If the given number between 100 and 400 is divisible by prime number less than 20 *i.e.*, 2, 3, 5, 7, 11, 13, 17 and 19, then it is not prime, otherwise it is prime.

Example 12: Which of the following numbers are prime numbers?

(*i*) 193 (*ii*) 277 (*iii*) 357

Solution : (*i*) 193 is not divisible by any of the prime numbers 2, 3, 5, 7, 11, 13, 17 and 19. So, 193 is a prime number.

(*ii*) 277 is not divisible by any of the prime numbers 2, 3, 5, 7, 11, 13, 17 and 19, so, it is a prime number.

(*iii*) 357 is divisible by 3. Therefore, 357 is not a prime number.

EXERCISE 3.2

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Determine the largest prime number that you need to test as a divisor to check whether or not 3… prime number.
2. Test the divisibility of the following numbers by 2 :
   (*i*) 367146 (*ii*) 42990 (*iii*) 567298
   (*iv*) 34503 (*v*) 29601 (*vi*) 12964
3. Test the divisibility of the following numbers by 3 :
   (*i*) 73101 (*ii*) 16422 (*iii*) 74912
   (*iv*) 70201 (*v*) 493605 (*vi*) 82568

4. Test the divisibility of
   (*i*) 75642
   (*iv*) 215370
5. Test the divisibility of
   (*i*) 918
   (*iv*) 397084
6. Test the divisibility of
   (*i*) 569288
   (*iv*) 7328
7. Using divisibility test
   (*i*) 3345
   (*iv*) 72385
8. Test the divisibility c
   (*i*) 61809
   (*iv*) 90728
9. In each of the follow
   (*i*) 56 * 5
   (*iv*) 357 * 46
10. In each of the follow
    (*i*) 35 * 6
11. Which of the follow
    (*i*) 277
    (*v*) 331

COMMON FACTOR

Common Factors

Factors of 4 are 1, 2, and
Factors of 6 are 1, 2, 3,
1 and 2 are the factors o
They are called the comm

Example 13: What are t
Solution : The factors
The factors
Clearly, 1,

Example 14: What are
Solution : Factors of
Factors of
Factors of
Clearly, 1,

4. Test the divisibility of the following numbers by 6 :

(*i*) 75642 (*ii*) 65472 (*iii*) 54623
(*iv*) 215370 (*v*) 29046 (*vi*) 809728

5. Test the divisibility of the following numbers by 4 :

(*i*) 918 (*ii*) 57312 (*iii*) 163976
(*iv*) 397084 (*v*) 72138 (*vi*) 615024

6. Test the divisibility of the following numbers by 8 :

(*i*) 569288 (*ii*) 67152 (*iii*) 59348
(*iv*) 7328 (*v*) 965214 (*vi*) 827432

7. Using divisibility test, determine which of the following numbers are divisible by 5 and 10 :

(*i*) 3345 (*ii*) 2968 (*iii*) 56780
(*iv*) 72385 (*v*) 92965 (*vi*) 206174

8. Test the divisibility of the following numbers by 11 :

(*i*) 61809 (*ii*) 38016 (*iii*) 746928
(*iv*) 90728 (*v*) 56248 (*vi*) 60847

9. In each of the following numbers, replace * by the smallest digit to make it divisible by 9 :

(*i*) 56 * 5 (*ii*) 31 * 42 (*iii*) 5702 *
(*iv*) 357 * 46 (*v*) 587 * 1 (*vi*) * 2066

10. In each of the following numbers, replace * by the smallest digit to make it divisible by 11 :

(*i*) 35 * 6 (*ii*) 439 * 71 (*iii*) 86 * 72 (*iv*) 1723 * 4

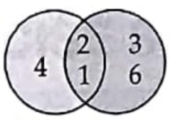
11. Which of the following are prime numbers?

(*i*) 277 (*ii*) 143 (*iii*) 253 (*iv*) 397
(*v*) 331 (*vi*) 167 (*vii*) 203 (*viii*) 161

COMMON FACTORS AND COMMON MULTIPLES

Common Factors

Factors of 4 are 1, 2, and 4.
Factors of 6 are 1, 2, 3, and 6.
1 and 2 are the factors of both 4 and 6.
They are called the common factors of 4 and 6.



Factors of 4 Factors of 6

Example 13: What are the common factors of 12 and 18?

Solution : The factors of 12 are 1, 2, 3, 4, 6, and 12.
The factors of 18 are 1, 2, 3, 6, 9, and 18.
Clearly, 1, 2, 3, 6 are the common factors of 12 and 18.

Example 14: What are the common factors of 4, 12 and 16?

Solution : Factors of 4 are 1, 2 and 4.
Factors of 12 are 1, 2, 3, 4, 6 and 12.
Factors of 16 are 1, 2, 4, 8 and 16.
Clearly, 1, 2 and 4 are the common factors of 4, 12 and 16.

Common Multiples

Let us consider two numbers 4 and 6.

The multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, ...

The multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, ...

We observe that 12, 24, 36, ... are multiples of both 4 and 6. These are called the common multiples of

Example 15: Find first three common multiples of 6 and 8.

Solution : Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, ...

Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72,... .

Thus, the first three common multiples of 6 and 8 are 24, 48 and 72.

PRIME FACTORISATION

A number can be written as the product of two of its factors. These factors may be prime or compo composite, the factors can be split up further till they are all prime numbers. Since a composite num be expressed as the product of prime numbers.

We write 18 = 3 × 6 and say 18 has been factorised. This is one of the factorisations of 18.

The others are

18 = 3 × 6 = 3 × 2 × 3

18 = 2 × 9 = 2 × 3 × 3

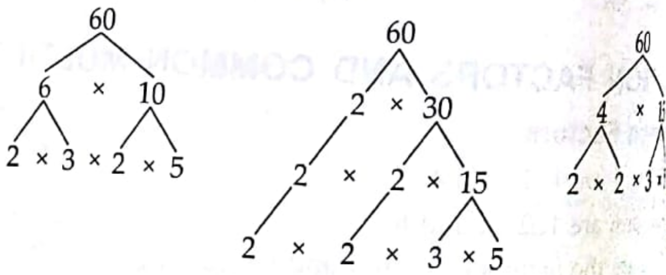
In both cases, we arrive at 2 × 3 × 3. In this factorisation, 2 and 3 are prime numbers. Such a factorisat a number is called *prime factorisation.*

This property is true for every composite number. Every composite number can be written as the produ prime factors in one and only one way except for the order of the factors. This is known as *fundam theorem of arithmetic.*

Factor Tree

Write the number you want to factorise at the top of the tree.

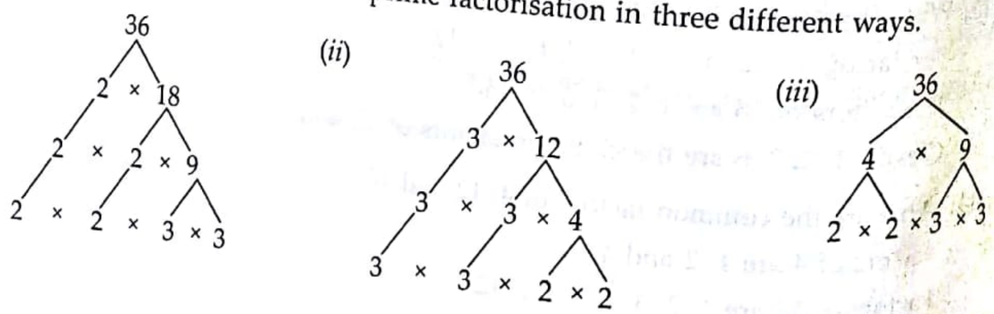
Choose any pair of factors as branches. If either of these is not prime, factorise again. Keep factorising until all the factors are prime.



Let us consider the number 60 and make factor tree.

Example 16: Make factor tree of 36 to write the prime factorisation in three different ways.

Solution : (*i*)



Division Method

We can also express a number as the product of primes by division method.

Let us find the prime factorisation of 216 by division method.

2	216
2	108
2	54
3	27
3	9
3	3
	1

$216 \div 2 = 108$

$108 \div 2 = 54$

$54 \div 2 = 27$

$27 \div 3 = 9$

$9 \div 3 = 3$

$3 \div 3 = 1$

Thus, $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Example 17: Express each of the following as the product of primes :

(*i*) 980 (*ii*) 8712

Solution : (*i*)

2	980
2	490
5	245
7	49
7	7
	1

Hence, $980 = 2 \times 2 \times 5 \times 7 \times 7$

(*ii*)

2	8712
2	4356
2	2178
3	1089
3	363
11	121
11	11
	1

Hence, $8712 = 2 \times 2 \times 2 \times 3 \times 3 \times 11 \times 11$

Example 18: I am the smallest number, having four different prime factors. Who am I?

Solution : The first four different prime numbers are 2, 3, 5, and 7.

Required number $= 2 \times 3 \times 5 \times 7 = 210$

EXERCISE 3.3

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Write the common factors of the following numbers :
   (*i*) 20, 28, 32 (*ii*) 15, 25, 30 (*iii*) 75, 60, 210
   (*iv*) 42, 63, 105 (*v*) 12, 18, 24
2. Write first three common multiples of 3, 5 and 6.
3. Write all the numbers less than 100 which are common multiples of 3 and 4.
4. Express each of the following numbers as the product of primes :
   (*i*) 420 (*ii*) 375 (*iii*) 945 (*iv*) 840
   (*v*) 637 (*vi*) 1080 (*vii*) 2904 (*viii*) 6000
   (*ix*) 1296 (*x*) 4725 (*xi*) 13915 (*xii*) 13500

HIGHEST COMMON FACTOR (H.C.F.)

The **Highest Common Factor** (H.C.F.) of two or more given numbers is the highest (or greatest) common factors. It is also known as ***Greatest Common Divisor*** (G.C.D.).

Let us find out the H.C.F. of 24 and 36.

Factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24.

Factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36.

Common factors of 24 and 36 are 1, 2, 3, 4, 6 and 12.

Highest among the common factors is 12 *i.e.*, H.C.F. = 12.

Prime Factorisation Method To Find H.C.F.

Step 1 : Find the prime factorisation of each of the given numbers.

Step 2 : Find the common prime factors of the given numbers.

Step 3 : The product of all common factors is the H.C.F. of the given numbers.

Example 19: Find the H.C.F. of the following numbers :

(*i*) 18, 48 (*ii*) 12, 16, 28 (*iii*) 70, 105, 175

Solution : (*i*)

2	18
3	9
3	3
	1

2	48
2	24
2	12
2	6
3	3
	1

$18 = \boxed{2} \times \boxed{3} \times 3$

$48 = \boxed{2} \times 2 \times 2 \times 2 \times \boxed{3}$

H.C.F. of 18 and 48 is $2 \times 3 = 6$.

(*ii*)

2	12
2	6
3	3
	1

2	16
2	8
2	4
2	2
	1

2	28
2	14
7	7
	1

$12 = \boxed{2} \times \boxed{2} \times 3$

$16 = \boxed{2} \times \boxed{2} \times 2 \times 2$

$28 = \boxed{2} \times \boxed{2} \times 7$

H.C.F. of 12, 16 and 28 is $2 \times 2 = 4$.

(*iii*)

2	70
5	35
7	7
	1

3	105
5	35
7	7
	1

5	175
5	35
7	7
	1

$70 = 2 \times \boxed{5} \times \boxed{7}$

$105 = 3 \times \boxed{5} \times \boxed{7}$

$175 = 5 \times \boxed{5} \times \boxed{7}$

H.C.F. of 70, 105 and 175 is $5 \times 7 = 35$.

Example 20: The H.C.F. of co-prime numbers 4 and 15 was found as follows by factorisation : $4 = 2 \times 2$ and $15 = 3 \times 5$ since there is no common prime factor, so H.C.F. of 4 and 15 is 0. Is the answer correct? If not, what is the correct H.C.F.?

Solution : H.C.F. of two or more numbers can never be zero, because 1 is the factor of every number.
Factors of 4 are 1, 2 and 4.
Factors of 15 are 1, 3, 5 and 15.
Thus, the H.C.F. of 4 and 15 is 1.

Example 21: Find the H.C.F. of 144, 180 and 192 by the prime factorisation method.

Solution : The numbers are 144, 180 and 192.

$$\begin{array}{r|l} 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 180 \\ \hline 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 192 \\ \hline 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$144 = \boxed{2} \times \boxed{2} \times 2 \times 2 \times \boxed{3} \times 3$

$180 = \boxed{2} \times \boxed{2} \times 3 \times \boxed{3} \times 5$

$192 = \boxed{2} \times \boxed{2} \times 2 \times 2 \times 2 \times \boxed{3}$

H.C.F. of 144, 180 and 192 is $2 \times 2 \times 3 = 12$.

EXERCISE 3.4

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

Find the H.C.F. of the given numbers using prime factorisation method :

1. 170, 238
2. 272, 425
3. 18, 24, 32
4. 91, 175, 49
5. 924, 1463, 1925
6. 150, 140, 210
7. 396, 720, 1080
8. 144, 252, 630
9. 1440, 1800, 1920
10. 72, 108, 180

Division Method to Find H.C.F. (Euclid's Algorithm)

Euclid, a Greek Mathematician devised a method to find the H.C.F. of two or more than two numbers. This method is known as *Euclid's algorithm* or *division method* for finding the H.C.F. of numbers.

Step 1 : Identify the greater number.
Step 2 : Take the greater number as dividend and the smaller number as divisor.
Step 3 : Find the quotient and remainder.
Step 4 : Take the remainder as the new divisor and the divisor as the new dividend.
Step 5 : Continue the procedure till the remainder is zero.
Step 6 : The last divisor, for which the remainder is zero, is the required H.C.F.

Example 22: Determine the H.C.F. of 18 and 30.

Solution : Divide the larger number (*i.e.*, 30) by the smaller number (*i.e.*, 18).
The remainder is 12.
Again dividing the new dividend (*i.e.*, 18) by the remainder (*i.e.*, 12).
The remainder is 6.
Again, dividing the new dividend (*i.e.*, 12) by the remainder (*i.e.*, 6).
The remainder is 0.
The last divisor is the H.C.F.
Thus, 6 is the H.C.F. of 18 and 30.

18) 30 (1
−18
12) 18 (1
−12
last divisor → 6) 12 (2
−12
0

Example 23: Find the H.C.F. of the following numbers using division method :

(*i*) 1045, 1520 (*ii*) 2349, 2835, 3078

Solution : (*i*) The numbers are 1045 and 1520.

1045) 1520 (1
– 1045
475) 1045 (2
– 950
95) 475 (5
– 475
0

Thus, the H.C.F. of 1045 and 1520 is 95.

(*ii*) The numbers are 2349, 2835, and 3078.

Let us first find the H.C.F. of 2349 and 2835.

2349) 2835 (1
– 2349
486) 2349 (4
– 1944
405) 486 (1
– 405
81) 405 (5
– 405
0

The H.C.F. of 2349 and 2835 is 81.

Now, we will find the H.C.F. of 81 and 3078.

81 divides 3078 exactly.

38
81) 3078 (
– 243
648
– 648
0

H.C.F. of 81 and 3078 is 81.

Thus, the H.C.F. of 2349, 2835 and 3078 is 81.

Example 24: The length, breadth and height of a room are 825 cm, 675 cm and 450 cm respectively. Find t longest tape which can measure the three dimensions of the room exactly.

Solution : The length of the longest tape which can measure the given lengths = H.C.F. of 825, 675 and 4

Let us first find the H.C.F. of 825 and 675.

675) 825 (1
– 675
150) 675 (4
– 600
75) 150 (2
– 150
0

⇒ H.C.F. of 675 and 825 is 75.

Now, we will find the H.C.F. of 75 and 450.

75) 450 (6
– 450
0

H.C.F. of 75 and 450 is 75.

The H.C.F. of 825, 675 and 450 is 75.

Thus, the required length of the tape is 75 cm.

Example 25: Three tankers contain 403 litres, 434 litres and 465 litres of diesel respectively. Find the maximum capacity of a container that can measure the diesel of the three tankers exact number of times.

Solution : The maximum capacity of the container is the H.C.F. of 403 litres, 434 litres and 465 litres.

Let us first take 403 and 434.

```
403) 434 (1
    -403
     31) 403 (13
        -403
           0
```

H.C.F. of 403 and 434 is 31.

Now, we find the H.C.F. of 31 and 465.

```
31) 465 (15
   -465
      0
```

H.C.F. of 403, 434 and 465 is 31.

Thus, the maximum capacity of the container is 31 litres.

Example 26: What is the greatest number which divides 615 and 963 leaving remainder 6 in each case?

Solution : If we subtract the remainder from the numbers, we have, 615 – 6 = 609, 963 – 6 = 957.

The required number is the H.C.F. of 609 and 957.

```
609) 957 (1
    -609
     348) 609 (1
         -348
          261) 348 (1
              -261
                87) 261 (3
                   -261
                      0
```

Thus, the H.C.F. of 609 and 957 is 87.

615 and 963 when divided by 87 leave remainder 6 in each case.

Thus, the required number is 87.

Example 27: What is the greatest number that can divide 781 and 458 leaving remainders of 1 and 3, respectively?

Solution : If we subtract the remainders from the numbers 781 and 458, we have 781 – 1 = 780, 458 – 3 = 455.

The required number is the H.C.F. of 780 and 455.

```
455) 780 (1
    -455
     325) 455 (1
         -325
          130) 325 (2
              -260
                65) 130 (2
                   -130
                      0
```

H.C.F. of 455 and 780 is 65.

Thus, the required number is 65.

Laughter Time

Teacher	:	"Who can tell me what 7 times 6 is?"
Student	:	"It's 42!"
Teacher	:	"Very good!-And who can tell me what 6 times 7 is?"
Same student	:	"It's 24!"

Example 28: There are 136 apples, 170 mangoes and 255 oranges. These are to be arranged in heaps containing the same number of fruits. Find the greatest number of fruits possible in each heap. How many heaps are formed?

Solution : We have 136 apples, 170 mangoes and 255 oranges.
Greatest number of fruits possible containing same number of fruits in each heap is equal to H.C.F. of 136, 170 and 255.

```
136) 170 (1
    -136
    ----
     34) 136 (4
        -136
        ----
           0

34) 255 (7
   -238
   ----
    17) 34 (2
       -34
       ---
         0
```

∴ H.C.F. of 136, 170 and 255 is 17.
Thus, 17 fruits should be arranged in each heap.
Total number of fruits = 136 + 170 + 255 = 561

Number of heaps = $\frac{561}{17} = 33$

Thus, 33 heaps are formed each having 17 fruits.

Example 29: In a seminar, the number of participants in English, Hindi and Sanskrit are 42, 63 and 105 respectively. Find the minimum number of rooms required, if in each room the same number of participants are to be seated and all of them being in the same subject.

Solution : The number of participants in each room is the H.C.F. of 42, 63 and 105.

```
42) 63 (1
   -42
   ---
   21) 42 (2
      -42
      ---
        0

21) 105 (5
   -105
   ----
      0
```

∴ H.C.F. of 42, 63 and 105 is 21.
In each room maximum 21 participants can be seated.
Total number of participants = 42 + 63 + 105 = 210

∴ Number of rooms required = $\frac{210}{21} = 10$

Thus, 10 rooms are required.

EXERCISE 3.5

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Which of the following are co-primes ?
   (*i*) 12, 30 (*ii*) 8, 21 (*iii*) 54, 144 (*iv*) 161, 192
   (**Hint :** *Two numbers are co-primes if their H.C.F. is 1.*)
2. Using division method, find the H.C.F. of :
   (*i*) 398, 436, 542 (*ii*) 120, 144, 204 (*iii*) 2057, 3267
   (*iv*) 658, 940, 1128 (*v*) 276, 1242
3. Find the H.C.F. of 96, 240 and 336.
4. Find the greatest number which divides 75 and 54 and leaves remainders 3 and 6 respectively.
5. Find the greatest number which divides 228, 305 and 382, leaving remainders 3, 5 and 7 respectively.

6. Find the greatest number which divides 245 and 1029, leaving a remainder 5 in each case.
7. Three pieces of timber, 54 m, 36 m and 24 m long, have to be divided into planks of the same length. What is the greatest possible length of each plank?
8. Determine the length of the longest tape which can be used to measure exactly the lengths 6 m 58 cm, 9 m 40 cm and 11 m 28 cm.
9. A room measures 4.8 m and 5.04 m. Find the size of the longest square tile that can be used to tile the floor without cutting any tile.
10. Two buckets can hold 272 *l* and 425 *l* of water respectively. Find the capacity of the largest bucket that can be filled an exact number of times from each bucket.
11. The students of three sections of a class have to stand in rows. Each row has equal number of students. If there are 24, 36 and 60 students in three sections, find the maximum number of students in each row.

LOWEST COMMON MULTIPLE (L.C.M.)

The *Lowest Common Multiple* of two or more given numbers is the smallest of their common multiples. In other words, it is the smallest number which is exactly divisible by each one of the given numbers.

For example :

(*i*) Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, ...
Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48, 54, ...
The common multiples of 4 and 6 are 12, 24, 36, 48, ...
Lowest common multiple of 4 and 6 is 12.

(*ii*) Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, ...
Multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72, ...
Multiples of 12 are 12, 24, 36, 48, 60, 72, 84, ...
Common multiples of 4, 8 and 12 are 24, 48, 72, ...
Lowest common multiple of 4, 8 and 12 is 24.

Prime Factorisation Method to Find L.C.M.

We follow the following steps to find the L.C.M. by prime factorisation method :

Step 1 : Write the given numbers as the product of prime numbers.

Step 2 : Count the maximum number of times each factor appears.

Step 3 : The product of these factors is the L.C.M. of the given numbers.

Example 30 : Find the L.C.M. of the following numbers by prime factorisation method :

(*i*) 27, 81 (*ii*) 24, 36, 54 (*iii*) 12, 24, 36 (*iv*) 15, 20, 35

Solution : (*i*)

3	27
3	9
3	3
	1

3	81
3	27
3	9
3	3
	1

27 = 3 × 3 × 3

81 = 3 × 3 × 3 × 3

Here, 3 occurs maximum number of four times, this happens for 81.

Thus, L.C.M. is 3 × 3 × 3 × 3 = 81.

(*ii*)

2	24
2	12
2	6
3	3
	1

2	36
2	18
3	9
3	3
	1

2	54
3	27
3	9
3	3
	1

$24 = 2 \times 2 \times 2 \times 3$

$36 = 2 \times 2 \times 3 \times 3$

$54 = 2 \times 3 \times 3 \times 3$

Here, 2 occurs maximum number of three times, this happens for 24.

3 occurs maximum number of three times, this happens for 54.

Thus, L.C.M. is $(2 \times 2 \times 2) \times (3 \times 3 \times 3) = 8 \times 27 = 216$

(*iii*)

2	12
2	6
3	3
	1

2	24
2	12
2	6
3	3
	1

2	36
2	18
3	9
3	3
	1

$12 = 2 \times 2 \times 3$

$24 = 2 \times 2 \times 2 \times 3$

$36 = 2 \times 2 \times 3 \times 3$

Here, 2 occurs maximum number of three times, this happens for 24.

3 occurs maximum number of two times, this happens for 36.

Thus, L.C.M. is $(2 \times 2 \times 2) \times (3 \times 3) = 8 \times 9 = 72$.

(*iv*)

3	15
5	5
	1

2	20
2	10
5	5
	1

5	35
7	7
	1

$15 = 3 \times 5$

$20 = 2 \times 2 \times 5$

$35 = 5 \times 7$

Here, 2 occurs maximum number of two times and each of 3, 5 and 7 occurs maximum number of times only once.

Thus, L.C.M. is $2 \times 2 \times 3 \times 5 \times 7 = 420$.

EXERCISE 3.6

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

Find the L.C.M. of the following numbers by prime factorisation method :

1. 24, 18 **2.** 12, 15, 20 **3.** 21, 28, 36 **4.** 16, 20, 40 **5.** 84, 90, 120

6. 14, 24, 48 **7.** 9, 12, 24 **8.** 24, 48, 80 **9.** 121, 1331 **10.** 42, 56, 132

Division Method To Find L.C.M.

To find the L.C.M. of two or more numbers by division method, following steps are involved :

Step 1 : Arrange the given numbers in a row separated by commas.

Step 2 : Divide the given numbers by a prime number which exactly divides at least two of the given numbers and carry forward the numbers which are not divisible.

Step 3 : Write the quotient in each case below the number in the next row.

Step 4 : Repeat the process till no two of the given numbers are divisible by the same prime number.

Step 5 : The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers.

Example 31: Find the L.C.M. of

(*i*) 36, 45 and 60 (*ii*) 24, 28 and 196

Solution : (*i*)

2	36, 45, 60	36 and 60 are divisible by 2.
2	18, 45, 30	18 and 30 are divisible by 2.
3	9, 45, 15	All numbers are divisible by 3.
3	3, 15, 5	3 and 15 are divisible by 3.
5	1 5, 5	Both 5s are divisible by 5.
	1, 1, 1	

L.C.M. of 36, 45 and 60 is $2 \times 2 \times 3 \times 3 \times 5 = 180$

(*ii*)

2	24, 28, 196
2	12, 14, 98
7	6, 7, 49
	6, 1, 7

L.C.M. of 24, 28 and 196 = $2 \times 2 \times 7 \times 6 \times 7 = 1176$

Example 32: Find the L.C.M. of 108, 135 and 162 by division method.

Solution :

2	108, 135, 162
3	54, 135, 81
3	18, 45, 27
3	6, 15, 9
	2, 5, 3

L.C.M. of 108, 135 and 162 is $2 \times 3 \times 3 \times 3 \times 2 \times 5 \times 3 = 1620$

Now, we will discuss some applications of L.C.M. in solving practical problems.

Example 33: Find the least number which when divided by 6, 15 and 18 leaves remainder 5 in each case.

Solution : The smallest number divisible by 6, 15, and 18 is their L.C.M. The smallest number which when divided by 6, 15, and 18 leaving a remainder 5 in each case is (L.C.M. of 6, 15, and 18) + 5.

2	6, 15, 18
3	3, 15, 9
	1, 5, 3

L.C.M. of 6, 15, and 18 is $2 \times 3 \times 5 \times 3 = 90$

Thus, the required number = 90 + 5 = 95.

Example 34: Determine the greatest 3-digit number exactly divisible by 8, 10 and 12.

Solution : The number exactly divisible by 8, 10, and 12 is their L.C.M.

2	8, 10, 12
2	4, 5, 6
	2, 5, 3

L.C.M. of 8, 10, and 12 is $2 \times 2 \times 2 \times 5 \times 3 = 120$

$$120 \overline{)\,999\,}\;(8 \qquad \underline{-960} \qquad 39$$

The greatest 3-digit number is 999.

We find that when 999 is divided by 120, the remainder is 39.

The required number = $999 - 39 = 960$

Thus, the greatest 3-digit number which is exactly divisible by 8, 10, and 12 is 960.

Example 35: Find the smallest 3-digit number which is exactly divisible by 6, 8, and 12.

Solution : The number exactly divisible by 6, 8, and 12 is their L.C.M.

2	6, 8, 12
2	3, 4, 6
3	3, 2, 3
	1, 2, 1

L.C.M. = $2 \times 2 \times 3 \times 2 = 24$

$$24 \overline{)\,100\,}\;(4 \qquad \underline{-96} \qquad 4$$

The smallest 3-digit number is 100.

We find that when 100 is divided by 24, the remainder is 4.

The required number = $(100 - 4) + 24 = 120$

Thus, the smallest 3-digit number which is exactly divisible by 6, 8, and 12 is 120.

Example 36: In a morning walk, three persons step off together. Their steps measure 80 cm, 70 cm, 75 cm respectively. What is the minimum distance each should walk so that all can cover same distance in complete steps?

Solution : The distance covered by each one of them has to be the same as well as minimum. The required minimum distance each should walk would be the L.C.M. of the measures of their steps.

Thus, we find the L.C.M. of 80, 70 and 75.

2	80, 70, 75
5	40, 35, 75
	8, 7, 15

L.C.M. = $2 \times 5 \times 8 \times 7 \times 15 = 8400$

Thus, the required minimum distance is 8400 cm or 84 m.

Example 37: Three bells are ringing continuously at intervals of 30, 36 and 45 minutes respectively. At what time will they ring together again, if they ring simultaneously at 8 a.m.?

Solution : To find the time when they will ring together again, we have to find the L.C.M. of 30, 36, and 45.

2	30, 36, 45
3	15, 18, 45
3	5, 6, 15
5	5, 2, 5
	1, 2, 1

L.C.M. of 30, 36 and 45 is $2 \times 3 \times 3 \times 5 \times 2 = 180$

The bells will ring together again after 180 minutes.

The time 180 minutes (*i.e.*, 3 hours) after 8 a.m. = 11 a.m.

Thus, the bells will ring together again at 11 a.m.

Example 38: Find the smallest number which when reduced by 6 is divisible by 12, 15, 20 and 27.

Solution : The smallest number divisible by 12, 15, 20, and 27 is their L.C.M. The smallest number which when reduced by 6 is divisible by 12, 15, 20, and 27 is (L.C.M. of 12, 15, 20, and 27) + 6.

2	12,	15,	20,	27
2	6,	15,	10,	27
3	3,	15,	5,	27
5	1,	5,	5,	9
	1,	1,	1,	9

L.C.M. of 12, 15, 20, and 27 is $2 \times 2 \times 3 \times 5 \times 9 = 540$

Thus, the required number = $(540 + 6) = 546$.

EXERCISE 3.7

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. Determine the L.C.M. of the following numbers by division method :
   (*i*) 6, 8, 45 (*ii*) 9, 18, 21 (*iii*) 15, 20, 30
   (*iv*) 105, 315, 693 (*v*) 20, 24, 36 (*vi*) 12, 15, 36, 45
   (*vii*) 15, 16, 35, 36 (*viii*) 15, 24, 36 (*ix*) 180, 384, 144
   (*x*) 88, 66, 132 (*xi*) 21, 28, 36, 45 (*xii*) 102, 170, 136
2. Find the least number divisible by each of the numbers 15, 20, 24, 32 and 36.
3. What is the smallest number which is exactly divisible by 585 and 624?
4. Find the lowest natural number which when divided by 15, 20, 25 and 45 leaves a remainder 8 in each case.
5. What is the smallest 5-digit number which is exactly divisible by 72 and 108 ?
6. Find the least number of six digits which is exactly divisible by 15, 20 and 25.
7. Find the least number from which 26 is subtracted, the result is exactly divisible by 16, 24 and 36.
8. Four bells ring at intervals of 9, 12, 15 and 18 seconds respectively. They begin by ringing together. After how long will they all ring together again?

PROPERTIES OF H.C.F. AND L.C.M.

Following are the properties concerning H.C.F. and L.C.M. :

1. *The H.C.F. of the given numbers is always smaller than or equal to the given numbers.*
2. *The L.C.M. of the given numbers is always greater than or equal to the numbers.*
3. *The H.C.F. of two co-prime numbers is always 1. Conversely, if H.C.F. of two numbers is 1, then the two numbers are co-primes.*
4. *The L.C.M. of two prime numbers is equal to their product. The L.C.M. of two co-primes is also equal to their product.*
5. *The H.C.F. of the given numbers is always a factor of their L.C.M.*

   For example : Consider the numbers 8, 12, 16, and 36.

2	8,	12,	16,	36
2	4,	6,	8,	18
2	2,	3,	4,	9
3	1,	3,	2,	9
	1,	1,	2,	3

   L.C.M. of 8, 12, 16, and 36 is $2 \times 2 \times 2 \times 3 \times 2 \times 3 = 144$

   Their H.C.F. is clearly 4.

   H.C.F. (*i.e.*, 4) is a factor of L.C.M. (*i.e.*, 144).
6. *If 'a' and 'b' are two given numbers such that 'a' is a factor of 'b', then their H.C.F. = a and L.C.M. = b.*

   For example : Consider two numbers 6 and 24. We know that, 6 is a factor of 24.

   H.C.F. of 6 and 24 is 6. L.C.M. of 6 and 24 is 24.

Relationship between H.C.F. and L.C.M. of two Numbers

The product of the H.C.F. and L.C.M. of two numbers is equal to the product of the numbers.

Let a and b be two numbers then, $a \times b$ = L.C.M. × H.C.F.

or, L.C.M. $= \frac{a \times b}{\text{H.C.F.}}$ or, H.C.F. $= \frac{a \times b}{\text{L.C.M.}}$

> **Puzzle**
>
> A girl was asked to find the LCM of 3, 5, 12 and another number. But while calculating, she wrote ... instead of 12 and yet came with the correct answer. What could be the fourth number?

For example : Consider two numbers 24 and 36.

24 = 2 × 2 × 2 × 3

36 = 2 × 2 × 3 × 3

H.C.F. of 24 and 36 is 12.

L.C.M. of 24 and 36 is 72.

Their product = 24 × 36 = 864

Product of their H.C.F. and L.C.M. = 72 × 12 = 864

Hence, Product of two numbers = Product of their H.C.F. and L.C.M.

Example 39 : The H.C.F. of two numbers is 12 and their product is 2160. Find their L.C.M.

Solution : Product of two numbers = H.C.F. × L.C.M.

2160 = 12 × L.C.M.

⇒ L.C.M. $= \frac{2160}{12} = 180$

⇒ Thus, the L.C.M. of two numbers is 180.

Example 40 : The L.C.M. of two co-prime numbers is 221. If one of the numbers is 17, find the other number.

Solution : H.C.F. of two co-prime numbers is 1.

First number × second number = L.C.M. × H.C.F.

17 × second number = 221 × 1

second number $= \frac{221}{17} = 13$

⇒ Thus, the other number is 13.

Example 41 : Can two numbers have 16 as their H.C.F. and 294 as their L.C.M.? Give reason in support of your answer.

Solution : H.C.F. of two or more numbers is always a factor of their L.C.M. In other words, H.C.F. of given numbers must divide their L.C.M. exactly.

16 does not divide 294 exactly.

Thus, two numbers cannot have 16 as their H.C.F. and 294 as their L.C.M.

EXERCISE 3.8

Use **Cordova Smart Class Software** on the **smart board** in class to do Exercise.

1. For each of the following pairs of numbers, verify the relationship between H.C.F. and L.C.M. :
   (*i*) 64, 84 (*ii*) 48, 96 (*iii*) 27, 81
   (*iv*) 15, 35 (*v*) 117, 221
2. Find the H.C.F. and L.C.M. of the following numbers :
   (*i*) 60, 75 (*ii*) 234, 572 (*iii*) 861, 1353
3. The H.C.F. and L.C.M. of two numbers are 8 and 576 respectively. If one number is 64, find the other number.

4. The product of two numbers is 864. If their L.C.M. is 72, what is their H.C.F.?
5. The product of two numbers is 6400. If their H.C.F. is 16, find their L.C.M.
6. Can two numbers have 14 as their H.C.F. and 276 as their L.C.M.? Give reasons.
7. The H.C.F. of two numbers is 145 and their L.C.M. is 2175. If one of the numbers is 435, find the other number.
8. Find the H.C.F. and L.C.M. of 1224 and 1768.
9. The L.C.M. of two co-prime numbers is 4875. If one of the numbers is 75, find the other number.
10. The H.C.F. of 276 and 1242 is 138. Find their L.C.M.

FACTS TO REMEMBER

- A factor of a number is an exact divisor of that number.
- The multiples of a number are obtained by multiplying it by natural numbers 1, 2, 3, ...
- Numbers which have only two factors (1 and the number itself) are called prime numbers.
- Numbers having more than two factors are called composite numbers.
- 1 is a factor of every number and is the only number, which is neither prime nor composite.
- Two co-primes need not be both prime numbers.
- 2 is the only even prime number.
- The prime numbers that occur with a difference of 2 are called twin primes.
- Co-prime numbers are those which have no common factors except one.
- The Highest Common Factor (H.C.F.) of two or more given numbers is the highest (or greatest) of their common factors.
- The Lowest Common Multiple of two or more given numbers is the smallest of their common multiples.
- L.C.M. of any two prime or co-prime numbers equal to their product.
- H.C.F. of two or more than two numbers is a factor of their L.C.M.
- If the sum of all the factors of a number is two times the number, then the number is called a perfect number.
- Every composite number can be written as the product of prime factors in one and only one way except for the order of the factors.
- A number is divisible by :
  (*i*) 2, if its ones digit is either 0, 2, 4, 6 or 8.
  (*ii*) 3, if the sum of the digits is divisible by 3.
  (*iii*) 4, if the last two digits are 0s or if the number formed by the last two digits (*i.e.*, ones and tens) is divisible by 4.
  (*iv*) 5, if its ones digit is either 0 or 5.
  (*v*) 6, if it is divisible by 2 and 3 both.
  (*vi*) 8, if the number formed by last three digits is divisible by 8.
  (*vii*) 9, if the sum of the digits is divisible by 9.
  (*viii*) 10, if its ones digit is 0.
  (*ix*) 11, if the difference of the sum of the digits at odd places and the sum of the digits at even places is either 0 or a multiple of 11.

HOTS QUESTIONS

1. Find the sum of all even numbers between 20 and 70 using suitable rearrangement.
2. 136x5785 is an 8-digit number exactly divisible by 15. What is the least possible value of digit *x*?

ANSWERS

Exercise 3.1

1. (*ii*), (*iii*), (*v*). The odd numbers end in 1, 3, 5, 7 or 9.
2. (*i*) 39, 52, 65, 78, 91, 104 (*ii*) 68, 85, 102, 119
3. 23, 29, 31
4. (*i*) Yes (*ii*) Yes (*iii*) No (*iv*) Yes
5. (*i*) 3 + 2 (*ii*) 5 + 7 (*iii*) 7 + 29 or 13 + 23 or 17 + 19
6. (*i*) 1, 3, 5, 9, 15, 45
   (*ii*) 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
   (*iii*) 1, 2, 3, 6, 13, 26, 39, 78
7. 32, 34, 36, 38, 40, 42, 44, 46, 48, 50
8. 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99
9. (*i*) 3 + 5 + 13 or 3 + 7 + 11
   (*ii*) 5 + 7 + 19 or 3 + 11 + 17 or 7 + 11 + 13
   (*iii*) 5 + 7 + 41 or 3 + 7 + 43 or 5 + 17 + 31
10. (*i*) 13 + 31 or 7 + 37 or 41 + 3
    (*ii*) 5 + 13 or 7 + 11
    (*iii*) 19 + 5 or 7 + 17 or 11 + 13

Exercise 3.2

1. 19, 397 is a prime number.
2. (*i*), (*ii*), (*iii*), (*vi*)
3. (*i*), (*ii*), (*v*)
4. (*i*), (*ii*), (*iv*), (*v*)
5. (*ii*), (*iii*), (*iv*), (*vi*)
6. (*i*), (*ii*), (*iv*), (*vi*)
7. (*i*), (*iii*), (*iv*), (*v*) are divisible by 5 and (*iii*) is divisible by 10
8. (*i*), (*ii*), (*iv*) are divisible by 11
9. (*i*) 2 (*ii*) 8 (*iii*) 4 (*iv*) 2 (*v*) 6 (*vi*) 4
10. (*i*) 8 (*ii*) 5 (*iii*) 3 (*iv*) 0
11. (*i*), (*iv*), (*v*), (*vi*)

Exercise 3.3

1. (*i*) 1, 2, 4 (*ii*) 1, 5 (*iii*) 1, 3, 5, 15 (*iv*) 1, 3, 7, 21 (*v*) 1, 2, 3, 6
2. 30, 60, 90
3. 12, 24, 36, 48, 60, 72, 84, 96
4. (*i*) 2 × 2 × 3 × 5 × 7 (*ii*) 3 × 5 × 5 × 5
   (*iii*) 3 × 3 × 3 × 5 × 7 (*iv*) 2 × 2 × 2 × 3 × 5 × 7
   (*v*) 7 × 7 × 13 (*vi*) 2 × 2 × 2 × 3 × 3 × 3
   (*vii*) 2 × 2 × 2 × 3 × 11 × 11
   (*viii*) 2 × 2 × 2 × 2 × 3 × 5 × 5 × 5
   (*ix*) 2 × 2 × 2 × 2 × 3 × 3 × 3 × 3
   (*x*) 3 × 3 × 3 × 5 × 5 × 7 (*xi*) 5 × 11 × 11 × 23
   (*xii*) 2 × 2 × 3 × 3 × 3 × 5 × 5 × 5

Exercise 3.4

1. 34 2. 17 3. 2 4. 7
5. 77 6. 10 7. 36 8. 18
9. 120 10. 36

Exercise 3.5

1. (*ii*) and (*iv*)
2. (*i*) 2 (*ii*) 12 (*iii*) 121 (*iv*) 94 (*v*) 138
3. 48 4. 24 5. 75
6. 16 7. 6 m 8. 94 cm
9. 24 cm 10. 17 litres 11. 12

Exercise 3.6

1. 72 2. 60 3. 252 4. 80
5. 2520 6. 336 7. 72 8. 240
9. 1331 10. 1848

Exercise 3.7

1. (*i*) 360 (*ii*) 126 (*iii*) 60 (*iv*) 3465 (*v*) 360 (*vi*) 180 (*vii*) 504 (*viii*) 360 (*ix*) 5760 (*x*) 264 (*xi*) 1260 (*xii*) 2040
2. 1440 3. 9360 4. 908 5. 10152
6. 100200 7. 170 8. 3 minutes

Exercise 3.8

2. (*i*) 15, 300 (*ii*) 26, 5148 (*iii*) 123, 9471
3. 72 4. 12 5. 400
6. No, H.C.F. of two or more numbers is always a factor of their L.C.M.
7. 725 8. 136, 15912 9. 65 10. 2484

HOTS

1. 1080 2. 1

MATHS LAB ACTIVITY

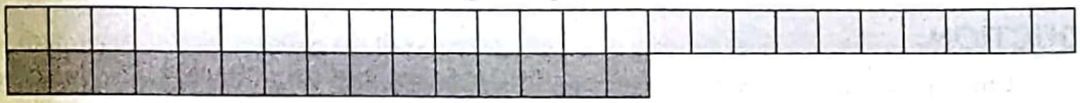
Objective : *To find the H.C.F. of two numbers by paper cutting and pasting.*

Materials required : Grid paper, a pair of scissors.

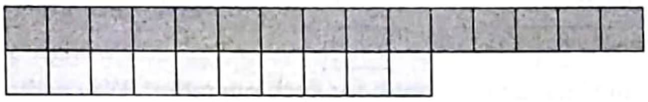
Activity : Let us find the H.C.F. of two numbers, 15 and 25.

Step 1 : Cut out two strips of length 25 cm and 15 cm from a grid paper which represents 25 and 15 respectively. Colour them dark blue and light blue respectively.

Step 2 : Place the smaller strip below the longer strip as shown below.

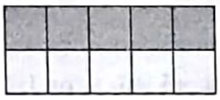


Step 3 : The difference of both the strips is 10 cm. Cut the extra length (*i.e.,* 25 cm – 15 cm = 10 cm) of dark blue strip and place it below the light blue strip of length 15 cm.



Step 4 : Cut the extra length (*i.e.,* 15 cm – 10 cm = 5 cm) of the light blue strip and place it below the dark blue strip of length 10 cm.

Step 5 : Cut the extra length (i.e., 10 cm – 5 cm = 5 cm) from the light blue strip and place it below the dark blue strip of length 5 cm.



Now, since both the strips are of equal length, so H.C.F. of 15 and 25 = 5.

Result : The HCF of two numbers is the length of one of the strips when both the strips become equal in the above process.

NCERT Exemplar Problems

Example : Fatima wants to mail three parcels to three village schools. She finds that the postal charges are ₹20, ₹28 and ₹36, respectively. If she wants to buy stamps only of one denomination, what is the greatest denomination of stamps she must buy to mail the three parcels?

Solution : The greatest denomination will be the H.C.F. of 20, 28 and 36.

By prime factorisation,

$$20 = \boxed{2} \times \boxed{2} \times 5$$
$$28 = \boxed{2} \times \boxed{2} \times 7$$
$$36 = \boxed{2} \times \boxed{2} \times 3 \times 3$$

∴ H.C.F. of 20, 28 and 36 = 2 × 2 = 4

Hence, the greatest denomination is ₹4.